HW2 - all exercises from [507], ch. 2

(i) $\frac{2}{3}, \frac{1}{3}, \frac{1}{2}$

(ii) nearly $\phi$, well below the minimum of the prob resulting from each individual action.

2.4 Fig. 2.18. All variables are $d$-connected to $A$.

Note that for variables adjacent to $A$, e.g., $C$, there is no path (chain) with an intermediate variable $V$ as required in Defn. 2.1 on p. 30 [507], so adjacent variable are not $d$-separated so they are $d$-connected.

Fig. 2.19. All except C and F. C and F are $d$-separated from $A$ because all paths between $A$ and $\{C, F\}$ go through $E \rightarrow I \rightarrow F$, and there is no evidence on $I$ or one of its descendants.
Each of the three pairs of adjacent vars cannot be 1-separated.

Fuel Meter Standing can be 1-separated from Start? by \{Fuel?, Start?, Clean Spark Plugs?\}

Fuel? can be 1-separated from Clean Spark Plugs by \{Start?, Fuel?\} and \{Fuel Meter Standing\}

Fig. 2.20

Find the hitting set of (unlabeled connections)

\[\{ A, F, E \}, \{ A, 0, B \}, \{ D, 3, B \}\]
2.6. \( C \) and \( E: \{A, B, D\} \) and \( \{B, D, F\} \)

\( A \) and \( B: \{\} \)

\( C \) and \( E: \{A, B, D, F\} \)

\( A \) and \( B: \{F\} \)

2.7

\( A: \{C, D, F\} \cup \{\} \cup \{A, D, F\} = \{A, C, D, E, F\} \)

\( B: \{C, E\} \cup \{\} \cup \{A, D, F\} = \{A, C, D, E, F\} \)

\( C: \{D\} \cup \{A, B\} \cup \{A\} = \{A, B, D\} \)

\( D: \{E\} \cup \{A, C\} \cup \{B, F\} = \{A, B, C, E, F\} \)

\( E: \{\} \cup \{B, D, E\} \cup \{\} = \{B, D, E\} \)

\( F: \{E\} \cup \{A\} \cup \{B, D\} = \{A, B, D, E\} \)
2.8 (Sketch) Consider all possible cases for chains ending at A:

1) Chain into a parent of A (and into A)
   blocked at the parent – serial connection

2) Chain out of a parent of A (and into A)
   blocked at the parent – diverging connection

3) Chain out of a child of A (and out of A)
   blocked at the child – serial connection

4) Chain into a child of A (and out of A)
   blocked at the spouse: serial or diverging
2.9. The ancestral graph of $A$, $C$, $B$ for the graph of Figure 2.19 is $\text{\begin{tikzpicture} \node (A) at (0,0) {A}; \node (B) at (1,0) {B}; \node (C) at (2,0) {C}; \end{tikzpicture}}$. There is no path from $A$ to $C$ in this graph, so there is no paths from $A$ to $C$ are intersected by $B$, so $A$ and $C$ are $d$-separated by $B$.

2.10. For the three networks (I means: $d$-separated; $\not\Box$ means: $d$-connected)

$\text{\begin{tikzpicture} \node (A) at (0,0) {A}; \node (B) at (1,0) {B}; \node (C) at (2,0) {C}; \end{tikzpicture}}$ $\begin{align*} A & \perp C \mid B \\
B & \perp C \mid A \end{align*}$ $\begin{align*} A & \perp C \mid \{B\} \\
B & \perp C \mid \{A\} \end{align*}$

so they are I-equivalent

For $\text{\begin{tikzpicture} \node (A) at (0,0) {A}; \node (B) at (1,0) {B}; \node (C) at (2,0) {C}; \end{tikzpicture}}$ $\begin{align*} A & \perp C \mid \{B\} \\
A & \perp C \mid \{B\} \end{align*}$

so this network is not I-equivalent to the other three.
2.12 \quad P(A), P(B),
\quad P(C \mid A, B)
\quad P(D \mid A, C)
\quad P(E \mid B, D, F)
\quad P(F \mid A)

2.14 \quad P(A, B, C, D, E, F) = P(A) P(B) P(C \mid A, B) P(D \mid A, C) P(E \mid B, D, F) P(F \mid A) =
\quad \sum_{E} \sum_{F} P(A) P(B) P(C \mid A, B) P(D \mid A, C) \sum_{E} P(E \mid B, D, F) P(F \mid A) =
\quad = P(A) P(B) P(C \mid A, B) P(D \mid A, C) \sum_{E} P(E \mid B, D, F) P(F \mid A) =
\quad = P(A) P(B) P(C \mid A, B) P(D \mid A, C) \sum_{F} P(F \mid A) =
\quad = P(A) P(B) P(C \mid A, B) P(D \mid A, C)
\[
\begin{align*}
P(B \mid D, A, c) &= \frac{P(A) P(B) P(c \mid A, B) P(D \mid A, c)}{\sum_B P(A) P(B) P(c \mid A, B) P(D \mid A, c)} \\
&= \frac{P(A) P(B) P(c \mid A, B) P(D \mid A, c)}{P(A) P(D \mid A, c)} = \frac{P(B, c \mid A)}{\sum_B P(B, c \mid A)} = \frac{P(B, c \mid A)}{P(c \mid A)}
\end{align*}
\]

Note how much easier it is to show that \( \{A, C\} \) separates \( B \) from \( D \).