Algorithm: satisfiability of a Horn formula

1. Mark every occurrence of an atomic formula $A$ in $F$ if there is a subformula of the form $(1 \rightarrow A)$ in $F$.

2. While there is a subformula $G$ in $F$ of the form $(A_1 \land \ldots \land A_n \rightarrow B)$ or of the form $(A_1 \land \ldots \land A_n \rightarrow 0)$, $n \geq 1$,
   where $A_1, \ldots, A_n$ are already marked (and $B$ is not yet marked),
If \( G \) is of the first form, then mark every occurrence of \( B \) else output "unsatisfiable" and halt.

3. Output "satisfiable" and halt. (The satisfying assignment is given by: \( A_i \) is true \( \iff A_i \) had a mark.)

a. This procedure is sound & complete? (proof similar to that of the bottom-up procedure for definite clauses)

b. Since "unsatisfiable" is output only for integrity constraints, every definite clause \( KB \) is satisfiable.

c. Sat. for Horn clauses is linear in the # of clauses.
a. The model obtained by this procedure is the minimal one. (Follows from the detailed proof: completeness part.)

b. $\text{KB} \models \neg a$

This has minimal model: empty (not the empty set is a model, $\emptyset$ is non-minimal)

Every Horn KB with no facts is satisfiable