

**CSCE 531 Spring 2008**  
**QUIZ 3**  
Assigned Friday, 08-01-25

Give a loop invariant for this program fragment :

```
x := 2;  
i := 1;  
(*What is the precondition here?*)  
while (i <= n) do  
  begin  
    x := x*x;  
    i := i+1  
  end
```

with precondition  $n \geq 1$  and postcondition  $x = 2^{2^n}$ .

**Answer:**  $x = 2^{2^{i-1}} \wedge i \leq n + 1$ .

Also answer the following questions.

1. What is the precondition before the loop? **Answer:**  $x = 2 \wedge i = 1 \wedge n \geq 1$
2. Your invariant should consist of the conjunction of two formulae. The second formula is:  $i \leq n + 1$ . Why is this formula needed? **Answer:** To insure that  $i = n + 1$  (rather than just  $i > n$ ) when the loop is exited.
3. Show that the precondition at the line with asterisks implies the invariant. **Answer:** In short: (1) Since  $i = 1$  and  $x = 2$ , then  $x = 2^{2^{i-1}}$ . (2) Since  $i = 1$  and  $n \geq 1$ , then  $i \leq n + 1$ .
4. Show that the invariant together with the negation of the loop implies the postcondition. **Answer:** In short: (1) Since  $i \leq n + 1$  and  $i > n$  then  $i = n + 1$ . (2) Since  $x = 2^{2^{i-1}}$  and  $i = n + 1$ , then  $x = 2^{2^n}$ .
5. Let  $x$  be the value of the variable  $x$  before executing the body of the loop and  $x'$  be the value of the variable  $x$  after executing the body of the loop. Write an equation that relates  $x$  and  $x'$ . **Answer:**  $x' = x * x$ .
6. Let  $i$  be the value of the variable  $i$  before executing the body of the loop and  $i'$  be the value of the variable  $i$  after executing the body of the loop. Write an equation that relates  $i$  and  $i'$ . **Answer:**  $i' = i + 1$ .