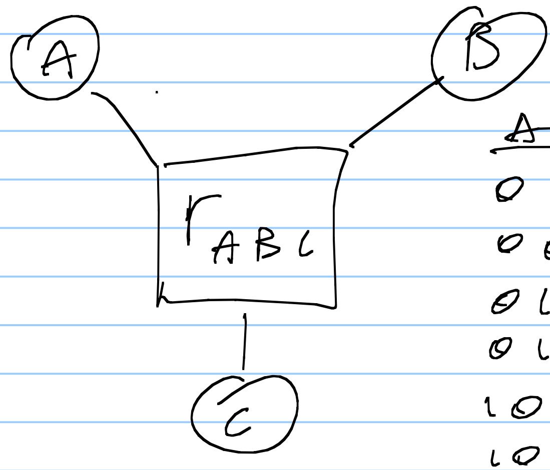
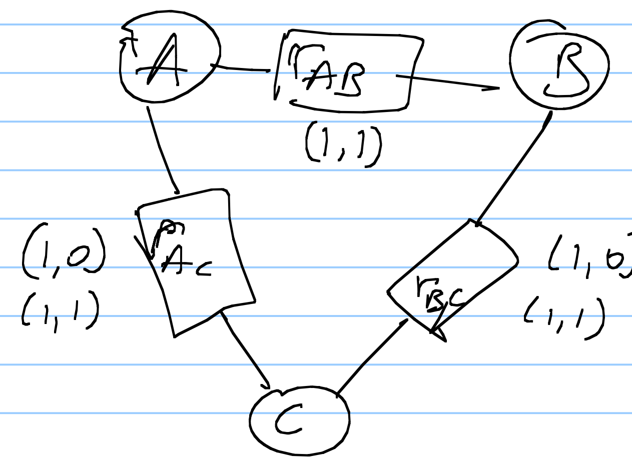
Review Session 580_2008_12_04

Note Title 12/4/2008

A, B, C are Boolean variables



A B C	r_{ABC}
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	0



(1, 1, 0) is in r_{ABC}

(1, 1, 1) is out of r_{ABC}

A triple is in r_{ABC} if exactly 2 of A, B, C have value 1.

$\Downarrow$

$\{011, 101, 110\} = r_{ABC}$

requires

$\{01, 10, 11\}$ in r_{AB}

$\{01, 11, 10\}$ in r_{AC}

$\{11, 01, 10\}$ in r_{BC}

This implies $111 \in r_{ABC}$, which is false.

Group axioms

$P(x, y, z) \equiv x \circ y = z$ (operation)

(examples of interpretation, have $\circ$ being $+$ or $\times$ and the reals)

(1) $\forall x \forall y \exists z \; P(x, y, z)$

(closure under $\circ$)

OR: $\forall x \forall y \exists z \; x \circ y = z$

(2) $\forall u \forall v \forall w \forall x \forall y \forall z$

$$((P(x, y, u) \wedge P(y, z, v)) \Rightarrow (P(x, v, w) \Leftrightarrow P(u, z, w)))$$

OR: $x \circ y = u \wedge y \circ z = v \Rightarrow (x \circ v = w \Leftrightarrow u \circ z = w)$

yz xy

(associate)

left-inverse of y

left-neutral element

(0 or 1 in the example interpretations)

(3) $\exists x \; (\forall y \; P(x, y, y) \wedge \forall y \exists z \; P(z, y, x))$

OR: $\exists x \; (x \circ y = y \wedge \exists z \; z \circ y = x)$

(existance of left-neutral element and of left-inverse)

Show the existence of right-inverses,

(4) $\exists x\,(\forall y\, P(x,y,y) \wedge \forall y\, \exists z\, P(\underline{y,z},x))$

$\underbrace{(1) \wedge (2) \wedge (3)}_{KB} \models (4) \iff (1) \wedge (2) \wedge (3) \wedge \sim(4)$ is inconsistent

$\{(1),(2),(3)\}$

We show the inconsistency by resolution

First, we convert the sentences (1), (2), (3), $\neg$(4) into clausal form and obtain (a), (b), (c), (d), (e), (f)

(a) comes from (1)

$\{P(x,y,m(x,y))\}$ (Skolemize (1)) (clauses are written as sets)

(b) $\{\neg P(x,y,u), \neg P(y,z,v), \neg P(x,v,w), P(u,z,w)\}$ come from (2), (b) for $\Rightarrow$

(c) $\{\neg P(x,y,u), \neg P(y,z,v), \neg P(u,z,w), P(x,v,w)\}$ (c) for $\Leftarrow$

(d) $\{P(e,y,y)\}$ (Skolemize part of (3)) (e is the name of the left-neutral element)

(e) $\{P(i(y),y,e)\}$ (Skolemize part of (3)) ($i(y)$ is the left-inverse of y)

(f) $\{\neg P(x, j(x), j(x)), \neg P(k(x), z, x)\}$

$\neg(x \circ j(x) = j(x) \wedge k(x) \circ z = x)$ Note that this claims that a left-inverse does not exist.

We should therefore expect our proof to work out!

(f) (d)

$\{\neg P(k(e), z, e)\}$

(b)

$\{\neg P(x, y, k(e)), \neg P(y, z, v), \neg P(x, v, e)\}$

(e)

$\{\neg P(i(v), w, k(e)), \neg P(w, z, v)\}$

(d)

$P(e, y, y)$ $\{x \backslash e,$ $P(e, y, y)$ $\{x \backslash e, y \backslash j(e)\}$ $P(e, j(e), j(e))$

$P(x, j(x), j(x))$ $\Longrightarrow$ $P(e, j(e), j(e))$ $\Longrightarrow$ $P(e, j(e), j(e))$

$$\{\neg P(i(v), e, k(e))\}$$

(c)

$$\{\neg P(i(t), y, w), \neg P(y, z, e), \neg P(u, z, k(e))\}$$

(d)

$$\{\neg P(i(t), y, e), \neg P(y, k(e), e)\}$$

(e)

$$\{\neg P(i(t), i(k(e)), e)\}$$

(f)

$$\Box$$

Another exercise with resolution refutation

(a) Every dragon is happy if all its children can fly

$\forall d\ (\forall x\ (Ch(x, d) \Rightarrow F(x)) \Rightarrow H(d))$

(b) Green dragons can fly

$\forall d\ Gr(d) \Rightarrow F(d)$

(c) A dragon is green if it is the child of at least one green dragon

$\forall d_1 (\exists d_2 (Ch(d_1, d_2) \wedge Gr(d_2)) \Rightarrow Gr(d_1))$

Show by res. ref. that it follows from (a), (b), (c) that all green dragons are happy.

(g) $\forall d (Gr(d) \Rightarrow H(d))$

$\sim$(g) $\sim \forall d\ (Gr(d) \Rightarrow H(d))$

$\sim \forall d\ \ (\sim Gr(d) \vee H(d))$

$\exists d \sim (\sim Gr(d) \vee H(d))$ $\rightarrow$ $\exists d\ \ (Gr(d) \wedge \sim H(d))$

$Gr(Dragon1) \wedge \sim H(Dragon1)$

$\boxed{\begin{array}{ll} (ng1) & Gr(Dragon1) \\ (ng2) & \sim H(Dragon1) \end{array}}$

(a) $\forall d \left[\left(\exists x \;\; Ch(x, d) \wedge \neg F(x) \right) \vee H(d) \right]$

$\forall d \; [\; Ch(Child(d), d) \wedge \neg F(Child(d)) \vee H(d)]$

Note: Ch is a predicate but Child is a function

(a1) $Ch(Child(d), d) \vee H(d)$

(a2) $\neg F(Child(d), d) \vee H(d)$

(b) $\sim Gr(d) \vee Fl(d)$

(c) . . .

$\sim Ch(d_1, d_2) \vee \sim Gr(d_2) \vee Gr(d_1)$

in implicative form, a Horn clause: green(D1) :- green(D_2), Ch(d_1, d_2) ✓

(a2) $\{\neg F(Child(d)), H(d)\}$

(ng2) $\{\sim H(Dragon1)\}$

(a1) $\{Ch(Child(d), d), H(d)\}$

$\{\neg F(Child(Dragon1))\}$

$\{Ch(Child(Dragon1), Dragon1)\}$

(b) $\{\sim Gr(d), F(d)\}$

$F(d)$ $\{d\}$ Child(Dragon1)$\}$

$F(Child(Dragon1))$

$\{\sim Gr(Child(Dragon1))\}$

(c) $\{\sim Ch(d_1, d_2), \sim Gr(d_2), Gr(d_1)\}$

$\{\sim Ch(Child(Dragon1), d2), \sim Gr(d2)\}$

$\{\sim Gr(Dragon1)\}$

(ng2) $\{Gr(Dragon1)\}$

$\square$