Give a loop invariant for this program fragment:

\begin{verbatim}
x := 2;
i := 1;
/*What is the precondition here?*/
while (i <= n) do
  begin
    x := x*x;
i := i+1
  end
\end{verbatim}

with precondition $n \geq 1$ and postcondition $x = 2^{2^n}$.

**Answer:** $x = 2^{2^{i-1}} \land i \leq n + 1$.

Also answer the following questions.

1. What is the precondition before the loop? **Answer:** $x = 2 \land i = 1 \land n \geq 1$

2. Your invariant should consist of the conjunction of two formulae. The second formula is: $i \leq n + 1$. Why is this formula needed? **Answer:** To insure that $i = n + 1$ (rather than just $i > n$) when the loop is exited.

3. Show that the precondition at the line with asterisks implies the invariant. **Answer:** In short: (1) Since $i = 1$ and $x = 2$, then $x = 2^{2^{i-1}}$. (2) Since $i = 1$ and $n \geq 1$, then $i \leq n + 1$.

4. Show that the invariant together with the negation of the loop implies the postcondition. **Answer:** In short: (1) Since $i \leq n + 1$ and $i > n$ then $i = n + 1$. (2) Since $x = 2^{2^{i-1}}$ and $i = n + 1$, then $x = 2^{2^n}$.

5. Let $x$ be the value of the variable $x$ before executing the body of the loop and $x'$ be the value of the variable $x$ after executing the body of the loop. Write an equation that relates $x$ and $x'$. **Answer:** $x' = x \times x$.

6. Let $i$ be the value of the variable $i$ before executing the body of the loop and $i'$ be the value of the variable $i$ after executing the body of the loop. Write an equation that relates $i$ and $i'$. **Answer:** $i' = i + 1$. 

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