CSCE 531 Spring 2007

Quiz 2

Assigned Wednesday, 07-01-24

Give a loop invariant for this program fragment :

```
x := 2;
i := 1;
(*What is the precondition here?*)
while (i <= n) do
  begin
    x := x*x;
    i := i+1
  end
```

with precondition $n \geq 1$ and postcondition $x = 2^{2^n}$.
Answer: $x = 2^{2^{i-1}} \wedge i \leq n+1$.
Also answer the following questions.

1. What is the precondition before the loop? **Answer:** $x = 2 \wedge i = 1 \wedge n \geq 1$

2. Your invariant should consist of the conjuction of two formulae. The second formula is: $i \leq n+1$. Why is this formula needed? **Answer:** To insure that $i = n+1$ (rather than just $i > n$) when the loop is exited.

3. Show that the precondition at the line with asterisks implies the invariant. **Answer:** In short: (1) Since $i = 1$ and $x = 2$, then $x = 2^{2^{i-1}}$. (2) Since $i = 1$ and $n \geq 1$, then $i \leq n+1$.

4. Show that the invariant together with the negation of the loop implies the postcondition. **Answer:** In short: (1) Since $i \leq n+1$ and $i > n$ then $i = n+1$. (2) Since $x = 2^{2^{i-1}}$ and $i = n+1$, then $x = 2^{2^n}$.

5. Let x be the value of the variable `x` before executing the body of the loop and x' be the value of the variable `x` after executing the body of the loop. Write an equation that relates x and x'. **Answer:** $x' = x * x$.

6. Let i be the value of the variable `i` before executing the body of the loop and i' be the value of the variable `i` after executing the body of the loop. Write an equation that relates i and i'. **Answer:** $i' = i + 1$.