2005 - 04 - 07

Note Title 4/7/2005

1. 9.3.2 Trace Dijkstra's algorithms on two graphs, p. 323 book, (a) and (b).

α) trace (a) using alg in handout - done earlier in class

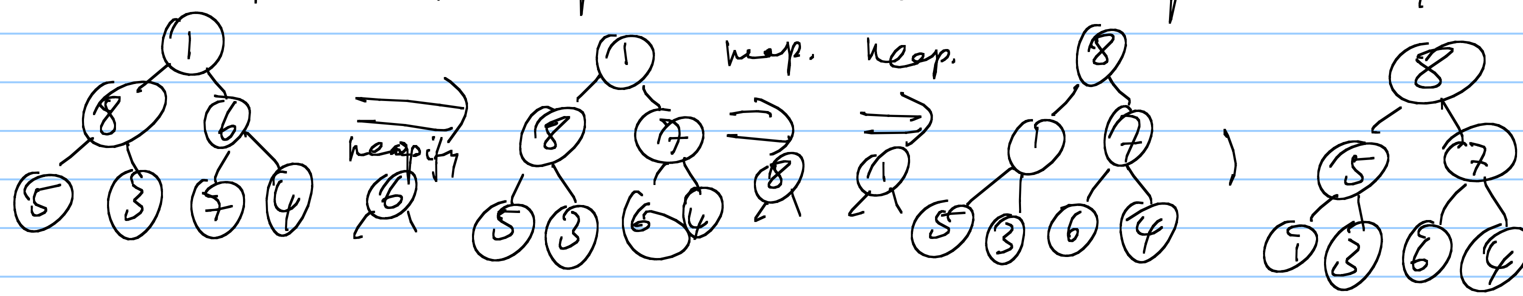
β) trace (b) using alg. in handout - choose a goal (target) node or continue until OPEN list is empty

γ) trace (b) using alg. on p. 323 of text.

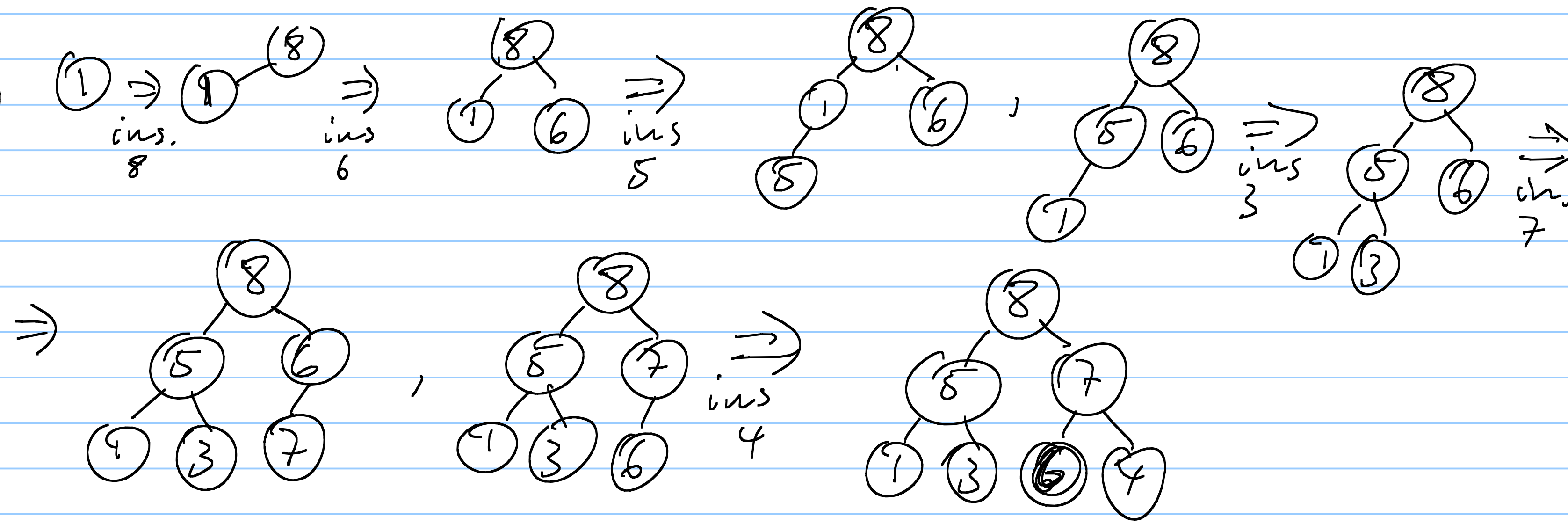
Note difference in author's solution between (a) and (b): in (a), g values are computed for all nodes; in (b), only for nodes in the OPEN list

2. 6.4.1 Heap construction. Construct heaps bottom-up (a), top-down (b) on input 1, 8, 6, 5, 3, 7, 4

(a)



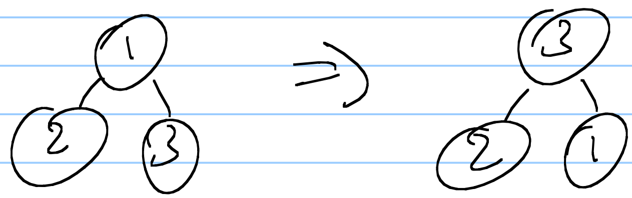
(b)



(c) Do top-down and bottom-up always yield the same heap?

No. Try input 1 2 3:

Bottom up:

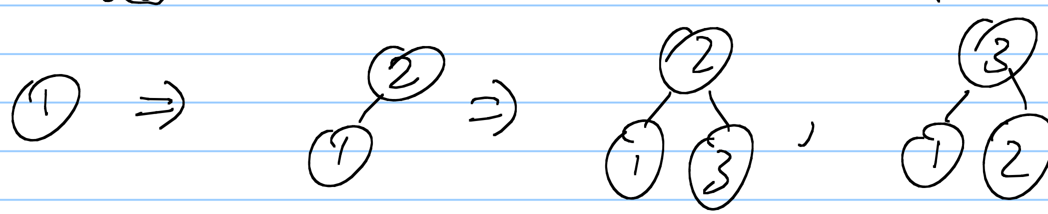


different!

Array representation

[3, 2, 1]

Top-down:



[3, 1, 2]

3. 8.3.10 : Matrix chain multiplication

Let us start with an example to describe the problem

A	×	B	×	C	×	D
30 × 1		1 × 40		40 × 10		10 × 25

$((AB)C)D$ (with AB: 30×40)

$$\underbrace{30 \times 40}_{AB} + \underbrace{40 \times 30 \times 10}_{(AB)C \; (30\times 40)(40\times 10)} + \underbrace{30 \times 10 \times 25}_{((AB)C)D \; (30\times 10)(10\times 25)} = 20{,}700 \quad !$$

$A[(BC)D]$

$$\underbrace{1 \times 40 \times 10}_{BC} + \underbrace{10 \times 25}_{(BC)D \; (1\times 10)(10\times 25)} + \underbrace{30 \times 25}_{A[(BC)D] \; (30\times 1)(1\times 25)} = 1{,}400$$

Define $M(i,j)$ = the min # of mult. needed to compute $A_i \times \cdots \times A_j$.

(d_{i-1}, d_i) (d_{k-1}, d_k) (d_k, d_{k+1}) (d_{j-1}, d_j)

$$(A_i \times \cdots \times A_k) \times (A_{k+1} \times \cdots \times A_j)$$

$$\begin{cases} M(i,j) = \min_{i \le k \le j-1} \left[M(i,k) + M(k+1,j) + d_{i-1} d_k d_j \right] \\ M(i,i) = 0 \end{cases}$$