Give a loop invariant for this program fragment:

\begin{verbatim}
x := 2;
i := 1;
while (i <= n) do
    begin
        x := x*x;
i := i+1
    end
\end{verbatim}

with precondition \( n \geq 1 \) and postcondition \( x = 2^{2^n} \).

**Answer:** \( x = 2^{2^{i - 1}} \land i \leq n + 1 \).

Also answer the following questions:

1. What is the precondition just before the loop (i.e., at the line with the comment)? **Answer:** \( x = 2 \land i = 1 \land i \leq n + 1 \).

2. Let \( x \) be the value of the variable \( x \) before executing the body of the loop and \( x' \) be the value of the variable \( x \) after executing the body of the loop. Write an equation that relates \( x \) and \( x' \). **Answer:** \( x' = x \times x \)

3. Let \( i \) be the value of the variable \( i \) before executing the body of the loop and \( i' \) be the value of the variable \( i \) after executing the body of the loop. Write an equation that relates \( i \) and \( i' \). **Answer:** \( i' = i + 1 \)

4. Your invariant should consist of the conjunction of two formulas. One of them should be very similar to the condition of the why loop. Explain why this formula is needed. **Answer:** To insure that \( i = n + 1 \), rather than just \( i > n \), when the loop is exited.

5. Show that the precondition at the line with the comment implies the invariant. **Answer:** In short: (i) since \( i = 1 \) and \( x = 2 \), then \( x = 2^{2^{i - 1}} \). (2) Since \( i = 1 \) and \( n \geq 1 \), then \( i \leq n + 1 \).

6. Show that the invariant together with the negation of the loop condition implies the postcondition. **Answer:** See quiz 7 fall 2007.