Product example (calculates product of a list of numbers)

Step 1, define the type

\[ \text{product} : [\text{Int}] \rightarrow \text{Int} \]

(we may generalize later; we "start simple")

Step 2, enumerate the case

\[ \text{product} \emptyset = (\text{empty list}) \]

\[ \text{product} \times : \times s = (\text{non-empty list}) \]
Step 3: Define the simple cases

\[ \text{product}(i) = 1 \]

\[ \text{product}(x \cdot x) = \]

(Usually, the simple cases become base cases, as it happens here.)
Step 4: Define the other case

\[ \text{product}([]) = 1 \]

\[ \text{product}(x \cdot xs) = x \cdot \text{product}xs \]

(As in this example, the “other” case often becomes the recursive case)
Step 5  Generalize and simplify

\[
\text{product} : \text{Num} \ a \Rightarrow [a] \to a
\]

The function definition is unchanged when we generalize from \text{Int} to any type of \text{Num} class.

\[
\text{foldr} \ (\ast) \ 1 \text{ when applied to } [a]
\]

\[
\text{the function that replaces cons (}: \text{cons}: \)
\]
foldr is actually very similar to
FP's `insert (!)`

`!` is the FP equivalent to
`foldr (\*) 1`

`!` works only on lists with at least two values

`foldr 1 (\*)` is actually closer to `!`. 
Homework: P 28. Exercise
6.3, 6.4, 6.5 (Ch 6 [4])

Please follow the hint for exercise 6.5.

$q_{sort}[5, 4, 3, 2, 1] = q_{sort}[5; 4, 3, 2, 1] = q_{sort}(\text{smaller}) + [5] + q_{sort}(\text{larger}) = q_{sort}[4, 3, 2, 1] + [5] + q_{sort}[1]$
\[ qsort(4;[3,2,1]) ++ [5] ++ [] = \]
\[ (qsort\ smaller ++ [4] ++ qsort\ larger) ++ [5] ++ [] = \]
\[ (qsort\ [3,2,1] ++ [4] ++ qsort\ [3]) ++ [5] ++ [] = \]
\[ (qsort\ [3;[2,1]]) ++ [4] ++ [3] ++ [5] ++ [] = \]
\[ (qsort\ [2,1] ++ [3] ++ qsort\ [3]) ++ [4] ++ [3] ++ [5] ++ [] = \]
\[ \text{qsort\_smaller} \to [2] \to \text{qsort\_larger} \to [3] \to [7] \to [6] \to \\
[3] \to [5] \to [7] \to \\
\text{qsort (1) \to (2) \to \text{qsort (3) \to (3) \to (7) \to (4) \to (7) \to (5) \to \\
(5) \to ]]} \to \\
\text{qsort (1; 3) \to (2) \to (3) \to (3) \to (7) \to (4) \to (7) \to (5) \to ]]} \to \\
[? = ] \to \\
\text{qsort\_smaller} \to [1] \to \text{qsort\_larger} \to [2] \to [7] \to [3] \to \\
\[
\]
\[
\]
\[
\vdots
\]
\[
\{1, 2, 3, 4, 15\}
\]