18.2 \[ T \]

\{ \text{Precondition} \} \rightarrow \{ \text{Postcondition} \}

If the precondition holds at the beginning of the program, then the postcondition holds at the end of the program. This computes the product of \( x \) and \( y \) \((x \# y)\) by repeated sums.
\{ y \geq 20 \} \implies z = 0 \quad \text{or} \quad \{ z = x + y \}$$

(a) Why is \( \{ y \geq 20 \land 0 = x \cdot (y-y) \} \) is equivalent to \( \{ y \geq 20 \} \)?

\[ \text{true, because any number multiplying by } \phi \text{ is } \phi \text{ and } y-y \text{ is zero for any value } y. \]

The condition \( \{ y \geq 20 \land 0 = x \cdot (y-y) \} \) may replace \( \{ y \geq 20 \} \) by precondition strengthening (precondition consequence rule).
(b) \[ y \geq 20 \land o = x / (y - y) \] 
assignment rule
\[ z = 0 ; \] 
sequence rule
\[ n = y ; \]
\[ y \geq 20 \land z = x \ (y - n) \land n \geq 20 \]

(c) \[ t(\delta) \]

Let \( y, n, z, x \) stand for their respective values before the execution of the loop body.
Let \( y', n', z', x' \) stand for the values of \( y, n, z, x \) after execution of the loop body.

So: \( y' = y \), \( z' = z + x \), \( n' = n - 1 \), \( x' = x \)
In short, you need to show

\[ A \cap T \Rightarrow T \]

\[
\begin{align*}
\{ \gamma \geq 0 \land n \geq 0 \land z = x(y - n) \land n \geq 0 \} & \Rightarrow \\
\{ \gamma' \geq 0 \land n' \geq 0 \land z' = x'(y' - n') \} & \Rightarrow \\
\gamma > 0 & \land n - 1 \geq 0 \equiv n \geq 1 \equiv (b/c \ n \ is \ integer) \equiv n \geq 0 \\
\gamma > 0 & \\
0.1 & \equiv x(y - n + 1) = x(y - n) + x.1
\end{align*}
\]

\[ z = x(y - n) \]
(e) In short, $I \wedge \neg T \Rightarrow Post$

\[ \{ y \geq 0 \land n \geq 0 \land z = x(y-n) \land \neg (n > 0) \} \Rightarrow z = x \# y \]

\[ n = 0 \]

\[ z = x(y-0) \Rightarrow z = x \# y \]