Def. 3.13 distinguishes discrete and continuous r.v.s.
Discrete r.v.s have discrete range; continuous r.v.s have continuous range.

Number of cards on top of a well-shuffled deck is an r.v. with a discrete (and finite) range: \([1..10]\).

1. The sum of the rolls of two dice is a discrete
(and finite) rv, because it has a discrete (and finite)
range, \([2, 12]\).

2. The number of arrivals at a website by time \(t\).
   This is a discrete. The range of this rv is the non-negative integers.

3. The time until the next arrival at a website.
   Continuous, at least in this course.
   (Computer 'time' functions give an integer value.)

4. The CPU requirement of an HTTP request.
   Continuous, with the same remark as \(3, \) but it is a time.
Example (2.1 [Trivial])

Consider a random experiment defined by three successive fair coin tosses. The sample space $\Omega$ for this experiment consists of a triple of 0s and 1s (or, if you prefer, heads and tails).

$\Omega = \{(000), (001), (010), (011), (100), (101), (110), (111)\}$

$|\Omega| = 8$

A possible rv, Max, maps each outcome to the largest value of $X_i$ in the outcome.

Max

\[
\begin{array}{cccc}
(000) & (001) & \cdots & (111) \\
0 & 1 & \cdots & 1 \\
\end{array}
\]
Another RV, $X$, maps each outcome to the number of ones (heads, successes) in the outcome.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>01</td>
<td>01</td>
<td>01</td>
<td>01</td>
</tr>
<tr>
<td>1</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>2</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>3</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td>00</td>
</tr>
</tbody>
</table>

Sample points $X$:

- $P(X = 0) = P\{s \in \Omega \mid X(s) = 0\}$
- $P\{0000\} = \frac{1}{8}$
- $P\{X = 1\} = P\{s \in \Omega \mid X(s) = 1\}$
- $P\{0110, 0100, 1000\} = \frac{3}{8}$
- $P\{X = 2\} = P\{s \in \Omega \mid X(s) = 2\}$
- $P\{011, 101, 110\} = \frac{3}{8}$
- $P\{X = 3\} = P\{s \in \Omega \mid X(s) = 3\}$
- $P\{111\} = \frac{1}{8}$
A r.v. (that represents a single coin flip) is called a Bernoulli random variable.

Def. 3.1.4 (p.m.f., probability mass function).

Let $X$ be a discrete r.v. Then, the p.m.f. of $X$ is defined as follows:

$$p_X(x) = P\{X = x\}, \quad \text{where} \quad \sum_{x} p_X(x) = 1$$

somes over the values (i.e. the range of the r.v. $X$).

The cumulative distribution function (c.d.f.) of $X$ is

$$F_X(x) = P\{X \leq x\} = \sum_{x \leq a} p_X(x)$$
Continuing the example of the r.v. sum of the successes in more fair coin tosses,

\[ F_X(0) = \Pr\{X \leq 0\} = \sum_{x \leq 0} p_X(x) = p_X(0) = \Pr\{X = 0\} = \Pr\{\text{tails}\} = \frac{1}{8} \]

\[ \Pr\{0\} = \frac{1}{8} \]

\[ F_X(1) = \Pr\{X \leq 1\} = \sum_{x \leq 1} p_X(x) = p_X(0) + p_X(1) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2} \]

\[ F_X(2) = \Pr\{X \leq 2\} = \sum_{x \leq 2} p_X(x) = p_X(0) + p_X(1) + p_X(2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8} \]

\[ F_X(3) = \Pr\{X \leq 3\} = \sum_{x \leq 3} p_X(x) = p_X(0) + p_X(1) + p_X(2) + p_X(3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1 \]
\[ F_X(a) = 1 - F_X(a) = P\{X > a\} = \sum_{x>a} f_X(x) \]

The Bernoulli r.v. is actually parametric. The parameter is the "coin bias."

A r.v. \( X \) is Bernoulli \((p)\) distributed (written \( X \sim \text{Bernoulli}(p) \)) if
\[
\begin{align*}
  f_X(1) &= p \quad (\text{success probability}) \\
  f_X(0) &= 1 - p \quad (\text{failure probability})
\end{align*}
\]