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Note Title 2016-02-02

Defn. 3.13 distinguishes discrete and continuous r.v.s.
(Discrete r.v.s have discrete range;
continuous rvs have continuous range)

Number value of card on top of a well-shuffled
deck is an r.v. with a discrete (and
finite) range: [1..10].

1. The sum of the rolls of two dice is a discrete

(and finite) rv, because it has a discrete (and finite) range: [2..12].

2. The number of arrivals at a website by time t
Discrete. The range of this rv is the non-negative integers

3. The time until the next arrival at a website
Continuous, at least in this course
(Computer "time" functions give an integer value.)

4. The CPU requirement of an HTTP request
Continuous, with the same remark as 3., b/c it is a time.

Example (2.1 [Trivedi])

Consider a random experiment defined by three successive fair coin tosses. The sample space Ω for this experiment consists of a triple of 0s and 1s, (or, if you prefer, heads and tails).

$\Omega = \{(000), (001), (010), (011), (100), (101), (110), (111)\}$

$|\Omega| = 8$

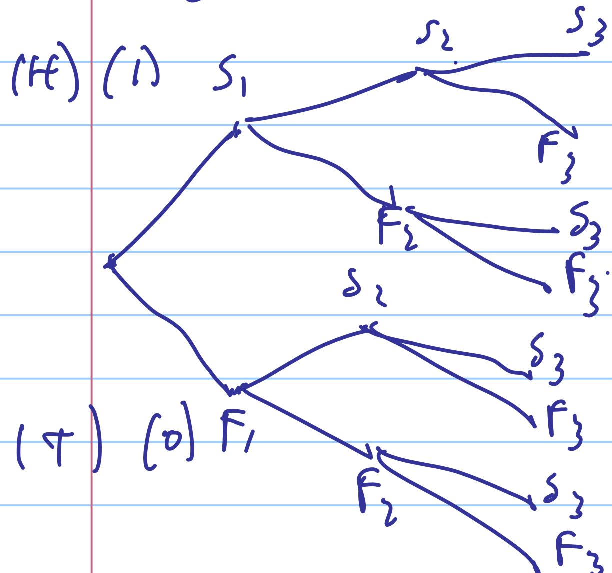
A possible rv, Max maps each outcome to the largest value in the outcome.

$$\begin{array}{lcccc} & (000) & (001) & \cdots & (111) \\ \text{Max} & \downarrow & \downarrow & & \downarrow \\ & 0 & 1 & & 1 \end{array}$$

Another rv, X, maps each outcome to the number of ones (heads, successes) in the outcome

	000	001	010	011	100	101	110	111
X	↓	↓	↓	↓	↓	↓	↓	↓
	0	1	1	2	1	2	2	3

"number of successes"



Sample points	X
111	3
110	2
101	2
100	1
011	2
010	1
001	1
000	0

$P(X=0) = P\{s \in \Omega / X(s)=0\}$,
$P\{000\} = \frac{1}{8}$

$P\{X=1\} = P\{s \in \Omega \mid X(s)=1\} =$
$= P\{001, 010, 100\} = \frac{3}{8}$

$P\{X=2\} = P\{s \in \Omega \mid X(s)=2\} =$
$= P\{011, 101, 110\} = 3/8$

$P\{X=3\} = P\{111\} = 1/8$

A r.v. that represents a single coin flip is called a Bernoulli random variable.

Defn. 3.14 (p.m.f., probability mass function).
Let X be a discrete r.v. Then, the pmf of X is defined as follows:

$$p_X(a) = P\{X = a\}, \text{ where } \sum_{x} p_X(x) = 1$$

x varies over the values (i.e. the range) of the r.v. X.

The cumulative distribution function (cdf) of X is

$$F_X(a) = P\{X \leq a\} = \sum_{x \leq a} p_X(x)$$

Continuing the example of the r.v sum of the successes in three fair coin tosses,

$$F_X(0) = P\{X \leq 0\} = \sum_{x \leq 0} p_X(x) = p_X(0) = P\{X=0\} = P\{000\} = \frac{1}{8}$$

$$\quad\quad\quad\quad\quad\quad \overset{\shortparallel}{P\{0\}} = \frac{1}{8}$$

$$F_X(1) = P\{X \leq 1\} = \sum_{x \leq 1} p_X(x) = p_X(0) + p_X(1) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$$

$$F_X(2) = P\{X \leq 2\} = \sum_{x \leq 2} p_X(x) = p_X(0) + p_X(1) + p_X(2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$$

$$F_X(3) = P\{X \leq 3\} = \sum_{x \leq 3} p_X(x) = p_X(0) + p_X(1) + p_X(2) + p_X(3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$$

We also write

$$\bar{F}_X(a) = 1 - F_X(a) = P\{X > a\} = \sum_{x > a} p_X(x)$$

The Bernoulli r.v. is actually parametric. The parameter is the "coin bias."

A r.v. X is Bernoulli(p) distributed (written $X \sim \text{Bernoulli}(p)$) if $p_X(1) = p$ (success probability)

$p_X(0) = 1-p$ (failure probability)