2.5 [H] Throughput & Utilization

\[
\lambda = \frac{1}{6}, \quad \mu = \frac{1}{3}
\]

\[
\lambda = \frac{1}{6}, \quad \mu = \frac{1}{5}
\]

Which of these has the higher throughput?

They are the same (\(x = \frac{1}{2}\))
Defn. Device utilization for device $i$, $C_i$, is the fraction of time that device $i$ is busy.

$$C_i = \frac{B}{\tau},$$
where $\tau$ is a long time interval and $B$ is the total busy time for device $i$ within interval $\tau$.

Device throughput for device $i$, $X_i$, is the rate of completions at device $i$ (jobs/sec).

\[\begin{bmatrix}
\text{A very special case:}
\end{bmatrix} \quad \begin{array}{c}
N = 1 \\
m_i = X_i
\end{array}
\]
$X_i = \frac{C}{T}$, where $T$ is a long time interval, and $C$ is the number of job completions within $T$.

Relationship between device utilization $C_i$ and device throughput $X_i$:

$X_i = \frac{C}{T} = \frac{C}{B} \cdot \frac{B}{T} = \frac{C}{B} \cdot \mu_i \cdot C_i = \frac{1}{\text{E}[S_i]} \cdot C_i$

$C_i = X_i \cdot \text{E}[S_i]$ Utilization $C_i$ on how many jobs

$1 \frac{\text{Jobs}}{\text{sec}} \cdot \frac{\text{sec}}{\text{jobs}}$
$x = \frac{1}{\mu}$, $\mu$ is given. What is $x$?

$C = \text{fraction of time the server is busy}$

$C = \frac{\text{average service time required by a job}}{\text{average time between arrivals}}$

$= \frac{1/\mu}{1/\lambda} = \frac{\lambda}{\mu} = \frac{\text{arrival rate}}{\text{service rate}}$

(proof is done)

$C = \text{fraction of time the server is busy}$

$X = \mathbb{E}[x] = \frac{d}{\mu}$, $\mu = d$. So, the throughput is not
Dependent on the service rate. The throughput for open
the simple one-queue, one-server system $iM0$
is the arrival rate. (1)

Closed Networks (2.6 [H])

Closed network $< \text{Batch}$

2.6.1 Interactive (Terminal-Driven) Systems
Diagram:

- **N user terminals**
- Central sub system
- Time spent here of Y
  - Think time, Z
- Response time is
  - Time between in and out
An individual user (e.g., call center operator) alternates between think state & submitted state.

\[ \text{Time in system (for a job)} = \text{sum of think time and response time} \]

\[ E[T] = E[R] + E[Z] \]

\( E[R] \) (between the input subsystem and output subsystem)
2.6.2 Batch Systems

\[ N \text{ user terminals} \]

Central subsystem

\[ \text{in} \quad \text{out} \]
\[ \text{NPL = N} \]

Single-server closed batch system:

\[ \mu = X = \frac{1}{E[S]} = \frac{1}{E[T]} \]
Tandem system (a closed bed system)

MPL = N