CSCE 317 Spring 2015
Final Exam
2015-04-30

Closed book and notes; X pages

Question 1 (15 points; textbook p.71))

Use the Inverse-Transform method to generate $X \sim Exp(\lambda)$.

Question 2 (20 points; textbook pp.107-108)

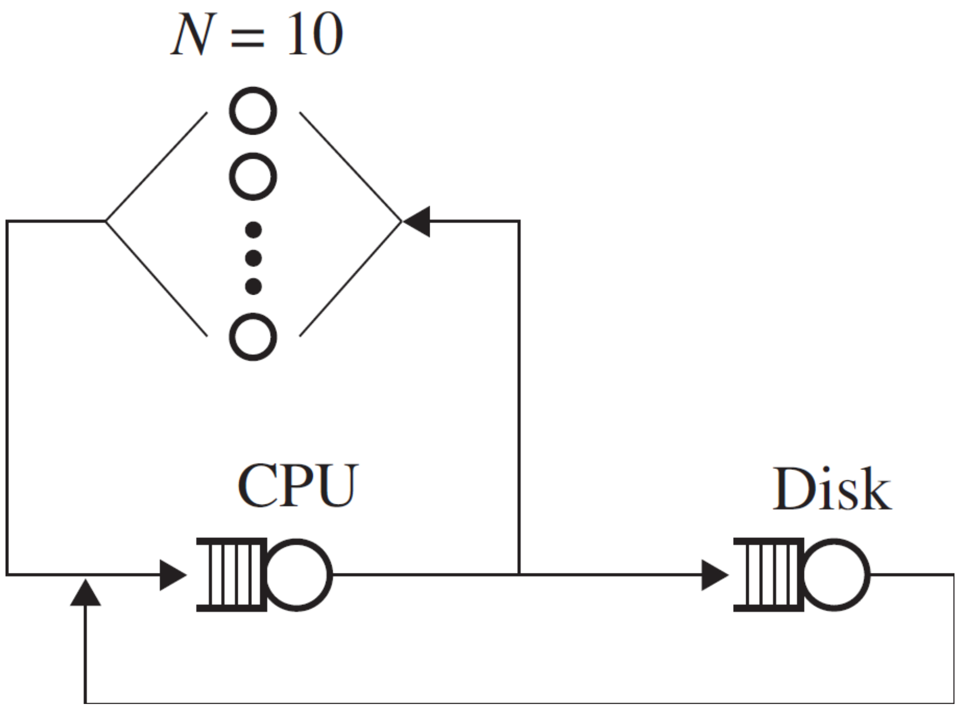
Suppose we have an interactive system with the following characteristics:

- 25 terminals ($N=25$)
- 18 seconds average think time ($E[Z]=18$)
- 20 visits to a specific per interaction on average ($E[V_{disk}]=20$)
- 30% utilization of that disk ($\rho_{disk}=0.3$)
- 0.025 sec average service time per visit to that disk ($E[S_{disk}]=0.025$)

What is the mean response time, $E[R]$?

Question 3 (15 points; textbook, pp.119-120)

Refer to the system in the Figure below, with N = 20 (not 10, as in the figure) and E[Z] = 5.



1. System A looks like the system in the figure above with D_{cpu}=4.6 and D_{disk}=4.0.
2. System B looks like the system in the figure above with D_{cpu}=4.9 and D_{disk}=1.9 (a slightly slower CPU and a much faster disk).

Which system has higher throughput? Compute N* for both systems, to insure that it is reasonable to use the "large N" approximation.

Question 4 (20 points; textbook, sections 8.3.1 and 8.7.1)

A machine M is either working or in the repair center. If it is working today, then there is a 95% chance that it will be working tomorrow. If it is in the repair center today, then there is a 40% chance that it will be working tomorrow.

(a) Name the two states for this repair facility problem.

(b) Draw the Markov chain diagram.

(c) The help desk is trying to figure out how much to charge to maintain machine M. They figure that it costs them $300 every day that machine M is in repair. What will the annual repair bill be?

Question 5 (25 points, textbook, exercise 9.3 [H])

Data centers alternate between "working" and "down." There are many reasons why data centers can be down, but for the purpose of this problem we mention only two reasons: (i) a backhoe accidentally dug up some cable, or (ii) a software bug crashed the machines. Suppose that a data center that is working today will be down tomorrow due to backhoe reasons with probability 1/6, but will be down tomorrow due to a software bug with probability ¼. A data center that is down today due to backhoe reasons will be up tomorrow with probability 1. A data center that is down today due to a software bug will be up tomorrow with probability ¾.

a) Draw a DTMC for this problem.
b) Is your DTMC ergodic? (You may assume that any finite-state, irreducible DTMC is positive recurrent, so you only need to show that the DTMC is aperiodic and irreducible.)
c) Is your DTMC time-reversible? Why or why not?
d) What fraction of time is the data center working?
e) What is the expected number of days between backhoe failures?

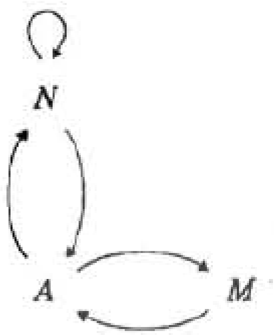
Question 6 (15 points; textbook, p.192)

Recall Google's (basic) PageRank Algorithm:

***Google's PageRank Algorithm*:**

1. Create a DTMC transition diagram where there is one state for each web page and there is an arrow from state i to state j if page i has a link to page j.
2. If page i has $k > 0$ outgoing links, then set the probability on each outgoing arrow from state i to be $1/k$.
3. Solve the DTMC to determine limiting probabilities. Pages are then ranked based on their limiting probabilities (higher probability first).

Suppose that the entire web consists of the three pages and links shown in the figure below.



(a) Annotate the figure with all link probabilities.
(b) Solve the DTMC to determine limiting probabilities.