317 2015-02-17

Note Title 2015-02-17

Example on p. 54 [H]

We computed the mean of the geometric distribution using the definition directly (p 45 [H]),

there is an easier way.

$N \sim \text{Geometric}(p)$

$E[N] = $ (the expected number of coin flips until the first head) $=$

$= \left(\sum_{n=1}^{\infty} n (1-p)^{n-1} p \right) =$ (condition on the value of the first flip) $=$

(*) $= \underbrace{E[N \mid Y=1]}_{1} \underbrace{P\{Y=1\}}_{p} + E[N \mid Y=0] P\{Y=0\} =$

(* Y is the value of the first flip) ("you wasted the first flip")

$$E[N] = 1p + [1 + E[N]](1-p)$$

$$p\,E[N] = p + 1 - p$$

$$E[N] = \frac{1}{p} \quad ✓$$

<u>Recall</u> Theorem 3.26 ("Linearity of Expectation"):

For r.v.s X and Y, $E[X+Y] = E[X] + E[Y]$.

Example (p 55 [H]): Binomial

$X \sim \text{Binomial}(n, p)$. What is $E[X]$?

$$E[X] = \sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} \qquad \text{Wow!}$$

Let X = number of successes in n trials = $X_1 + X_2 + \dots X_n$, where $X_i = \begin{cases} 1 & \text{if trial } i \text{ succeeds} \\ 0 & \text{otherwise} \end{cases}$

(We say that X_i is an indicator r.v.).

$E[X_i] = p$

$E[X] = E\left[\sum_{i=1}^{n} X_i\right] = E[X_1] + E[X_2] + \dots + E[X_n] = \underline{np}$ ✓

Example (p. 56 [H]): Hats

$X = I_1 + I_2 + \dots + I_n$, $I_i = \begin{cases} 1 & \text{if person } i \text{ gets his/her hat back} \\ 0 & \text{otherwise} \end{cases}$

These indicator variables are not independent; they are identically distributed.

still, linearity of expectation holds.

$$E[X] = E[I_1] + E[I_2] + \cdots + E[I_n] =$$

$$n \cdot E[I_i] = n\left(\frac{1}{n} \cdot 1 + \frac{n-1}{n} \cdot 0\right) = 1$$

Normal distribution