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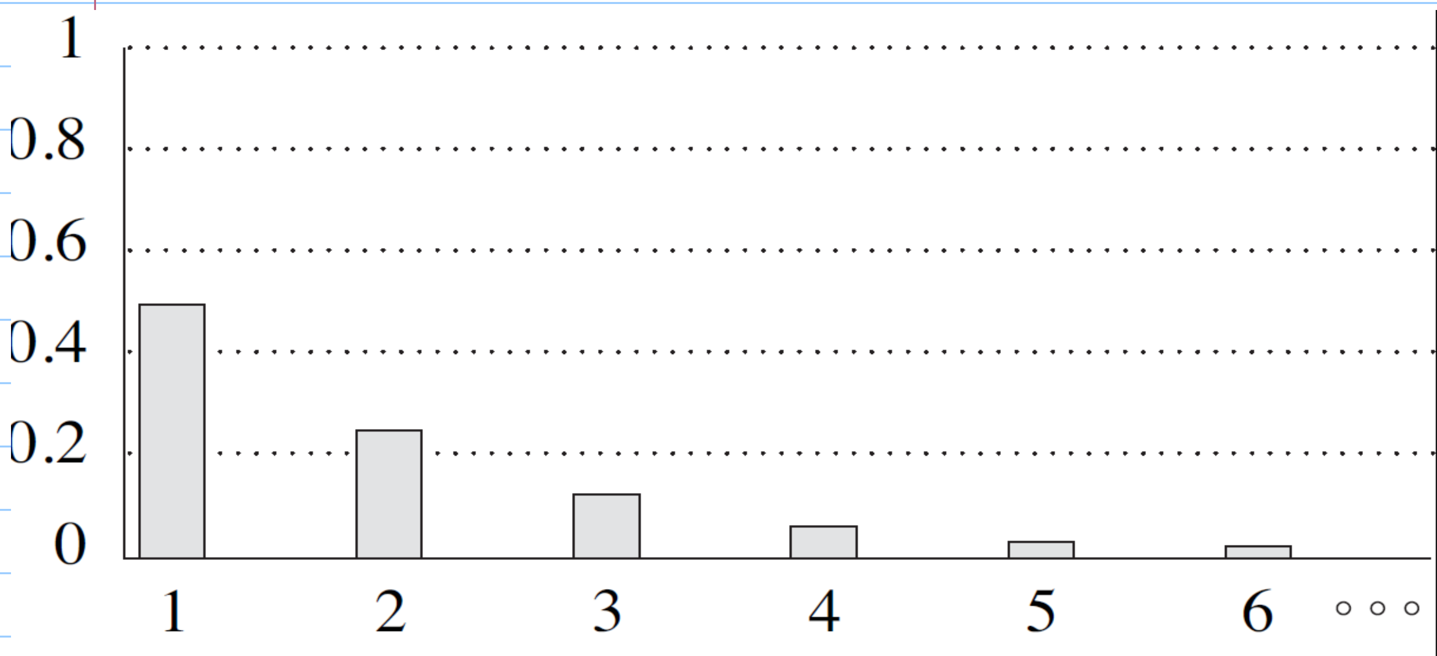
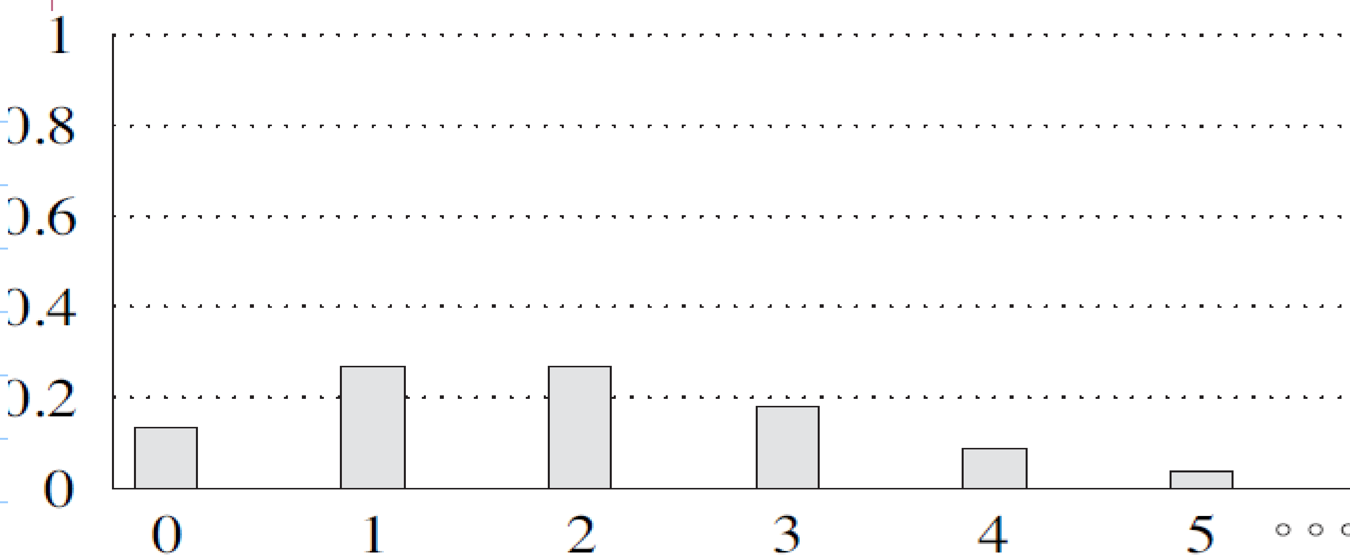
Note Title 2015-02-03



pmf of Geometric (0.5)



Poisson, $\lambda = 2$

Poisson(λ)

$$p_X(i) = \frac{e^{-\lambda}\lambda^i}{i!}, \quad i = 0, 1, 2, \ldots$$

Ex. Number of births that die in the first year can be approximated by Poisson (n p).

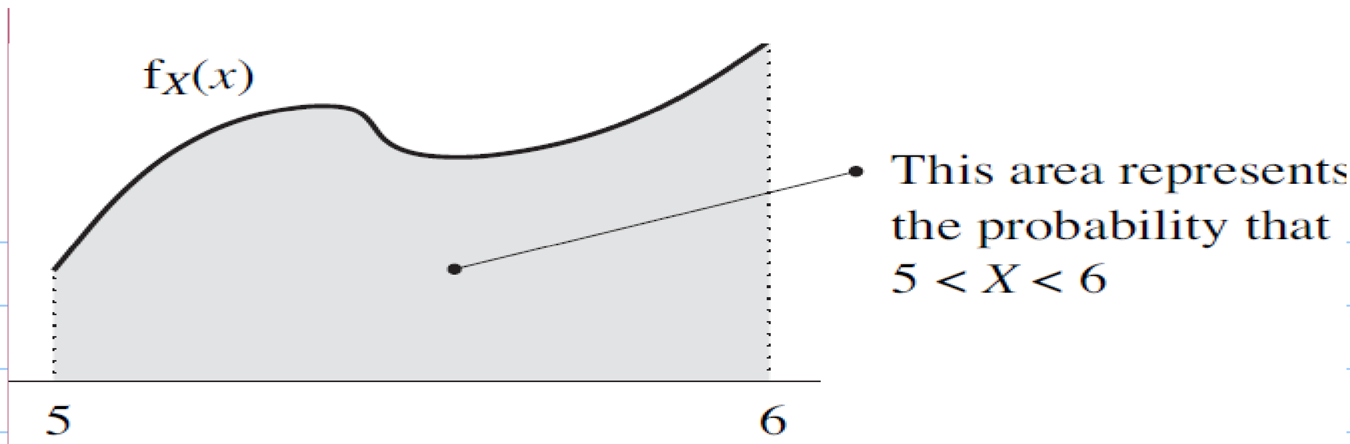
$\lambda = n \cdot p$

More precisely, it is Binomial (n, p)

Continuous r.v.

The p.d.f. of a continuous r.v. X is a non-negative function $f_X(\cdot)$, where

$P(a \le X \le b) = \int_a^b f_X(x)\,dx$ and where $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$



Ex. $f_X(x) = \begin{cases} .5x^{-.5} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty} dx = \int_0^1 .5x^{-.5}\, dx = .5\int_0^1 x^{-.5}\, dx = .5 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_0^1 = x^{\frac{1}{2}}\Big|_0^1 = 1$$

$f_X(x) = \begin{cases} 2x^{-2} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$$\int_{-\infty}^{\infty} dx = \int_0^1 2x^{-2}\,dx = 2 \cdot \frac{x^{-1}}{-1}\Big|_0^1 = \frac{-2}{x}\Big|_0^1 \text{ diverges}$$

$$f_X(x) = \begin{cases} x^{-2}, & 1 \le x < \infty \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} dx = \int_1^{\infty} x^{-2}\,dx = \frac{x^{-1}}{-1}\Big|_1^{\infty} = -\frac{1}{x}\Big|_1^{\infty} = 0 - (-1) = 1$$

The cumulative distribution function of a continuous r.v. X is the function $F(\cdot)$ defined by

$$F_X(a) = P\{-\infty < X \le a\} = \int_{-\infty}^{a} f_X(x)\,dx$$

Also, $\bar{F}(a) = 1 - F_X(a) = P\{X > a\}$

$$f_X(x) = \frac{d}{dx}\int_{-\infty}^{x} f(t)\,dt = \frac{d}{dx} F_X(x)$$