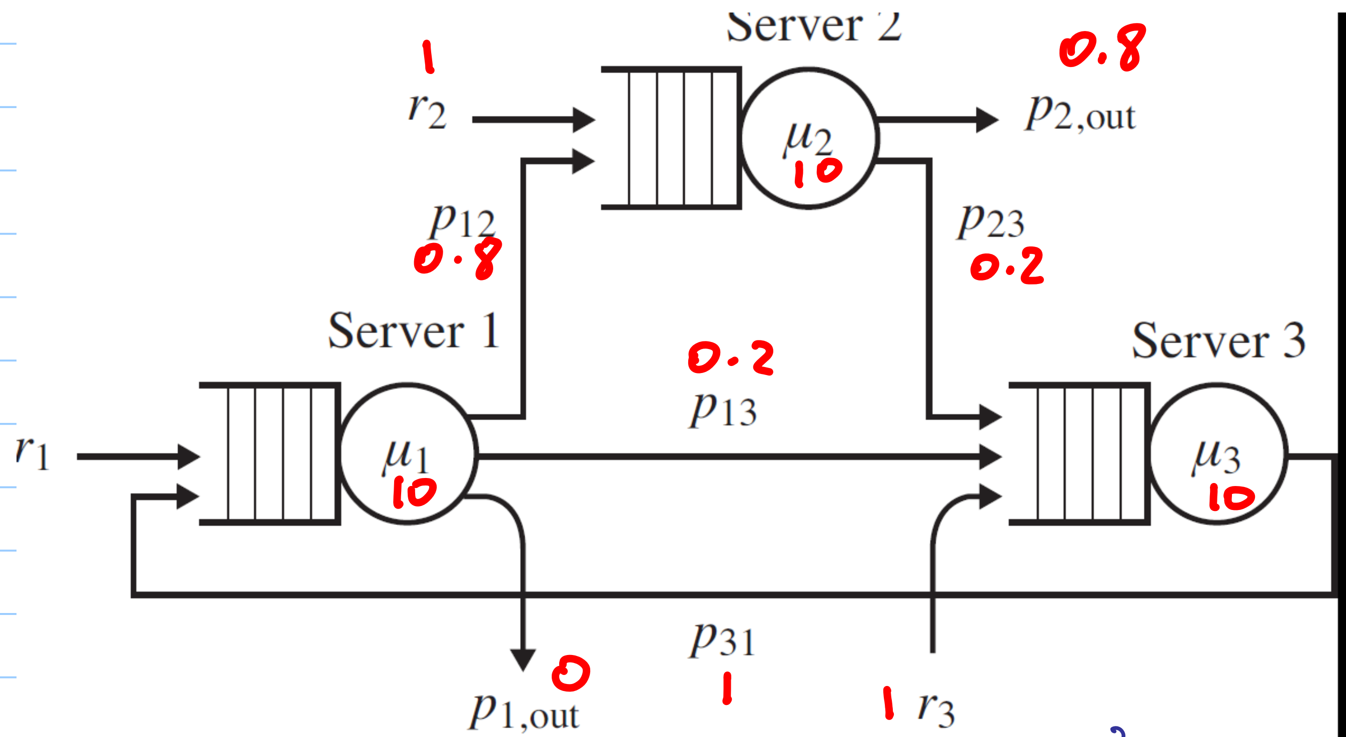
317 2015-01-20

Note Title 2015-01-20



The system throughput is $\sum_{i=1}^{3} r_i$ $(=X)$.

What is

The throughput at server i, X_i ?

$X_i = \lambda_i$ (total arrival rate at server i)

$\lambda_i = r_i + \sum_j \lambda_j P_{ji}$

In particular, $\lambda_i < \mu_i$ to have equilibrium at each server.

Exercise 2.1. Consider the open network above, Suppose $\mu_i = 10$, $\forall i$. Suppose the $r_2 = r_3 = 1$ Suppose that $p_{12} = 0.8 = p_{2.out}$, $p_{23} = p_{13} = 0.2$, $p_{1.out} = 0$, $p_{31} = 1$. What is the maximum value of

r_i to keep this system stable?

We start by writing the throughput equation for each server

$$
\begin{cases}
\lambda_1 = r_1 + \lambda_3 & \left(\lambda_1 = r_1 + \sum_{j=1}^{3} \lambda_j \cdot P_{j1}\right) \\
\lambda_2 = \cancel{r_2} + 0.8\,\lambda_1 & \left(\lambda_2 = r_2 + \sum_{j=1}^{3} \lambda_j \cdot P_{j2}\right) \\
\lambda_3 = \cancel{r_3} + 0.2\,\lambda_1 + 0.2\,\lambda_2 & \left(\lambda_3 = r_3 + \sum_{j=1}^{3} \lambda_j \cdot P_{j3}\right)
\end{cases}
$$

$$
\begin{bmatrix} 1 & 0 & -1 \\ -0.8 & 1 & 0 \\ -0.2 & -0.2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 = 1 \\ r_3 = 1 \end{bmatrix}
$$

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \text{inverse of} \cdot \begin{bmatrix} r_1 \\ 1 \\ 1 \end{bmatrix} \odot \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$$

n $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ μ

The inverse of A, A^{-1}, is the matrix whose elements are

$$a_{ij} = \frac{A_{ji}}{|A|}$$, where $|A|$ is the determinant of A

cofactor $\qquad$ minor

and A_{ji} is the determinant of the matrix obtained from A by removing row j and column i and multiplying by -1 if $i+j$ is odd.

$$|A| = \begin{vmatrix} 1 & 0 & -1 \\ .8 & 1 & 0 \\ -0.2 & -0.2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 0 \\ -.2 & 1 \end{vmatrix} - 0 + (-1) \begin{vmatrix} -.8 & 1 \\ -.2 & -.2 \end{vmatrix} =$$

$$= 1 - .36 = .64$$

$$A_{11} = \begin{vmatrix} 1 & 0 \\ -0.2 & 1 \end{vmatrix} = 1$$

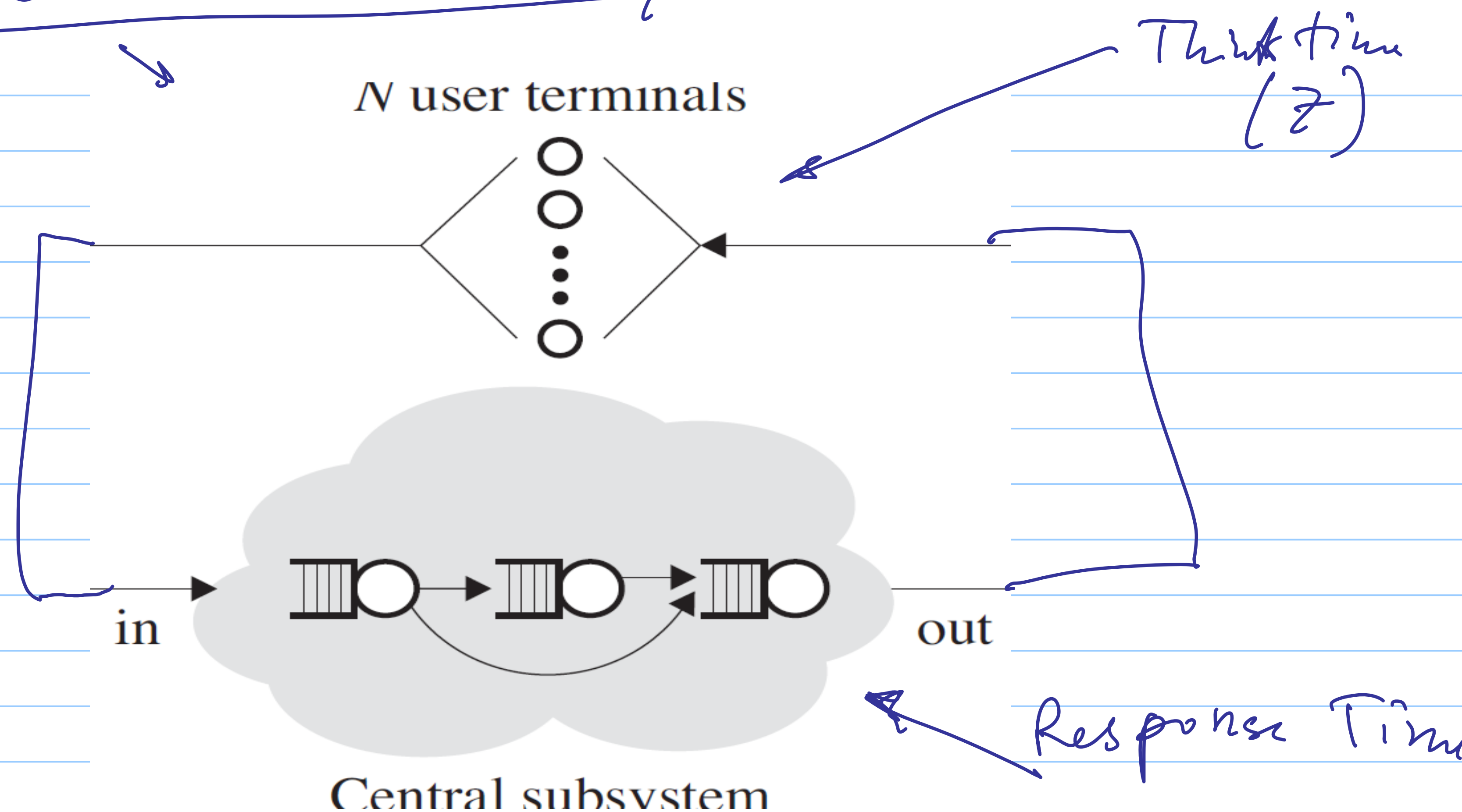
$$A_{21} = \begin{vmatrix} 0 & -1 \\ -0.2 & 1 \end{vmatrix} \times (-1) = 0.2$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \overset{\frac{1}{64}}{\frac{1}{16}} \begin{bmatrix} 25 & 5 & 25 \\ 20 & 20 & 20 \\ 9 & 5 & 25 \end{bmatrix} \begin{bmatrix} r_1 \\ 1 \\ 1 \end{bmatrix} < \underset{\underline{M}}{\begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}}$$

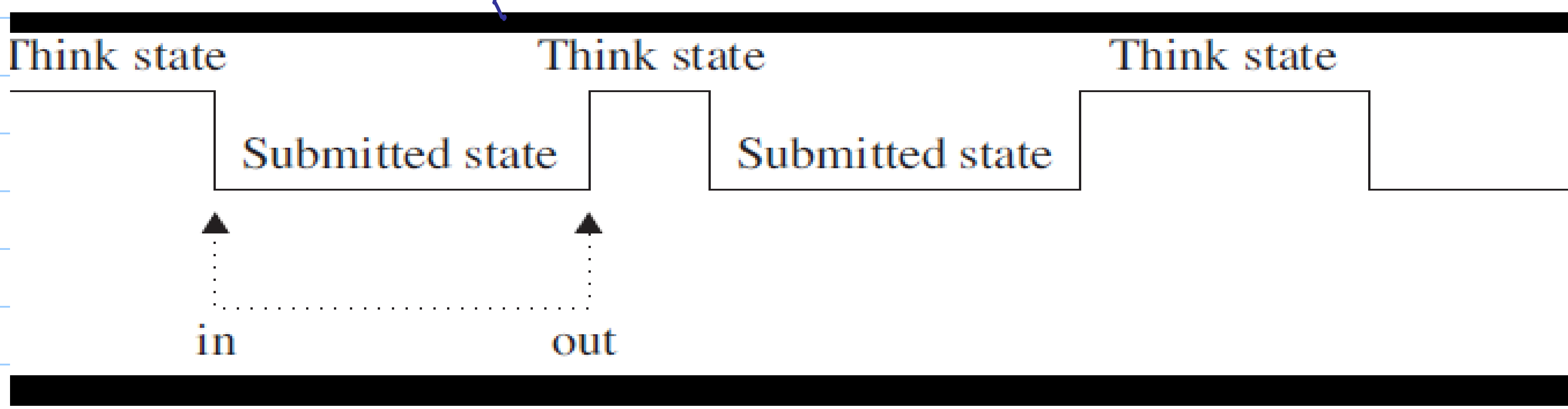
$$\begin{cases} \frac{1}{16}(25 r_1 + 5 + 25) < 10 & r_1 < \frac{130}{25} = \frac{26}{5} \\ \frac{1}{16}(20 r_1 + 20 + 20) < 10 & r_1 < 6 \\ \frac{1}{16}(9 r_1 + 5 + 25) < 10 & r_1 < \frac{130}{9} \end{cases} \Rightarrow$$

$$\Rightarrow \quad r_1 < \frac{26}{5}$$

Closed interactive system



State alternation for an individual user



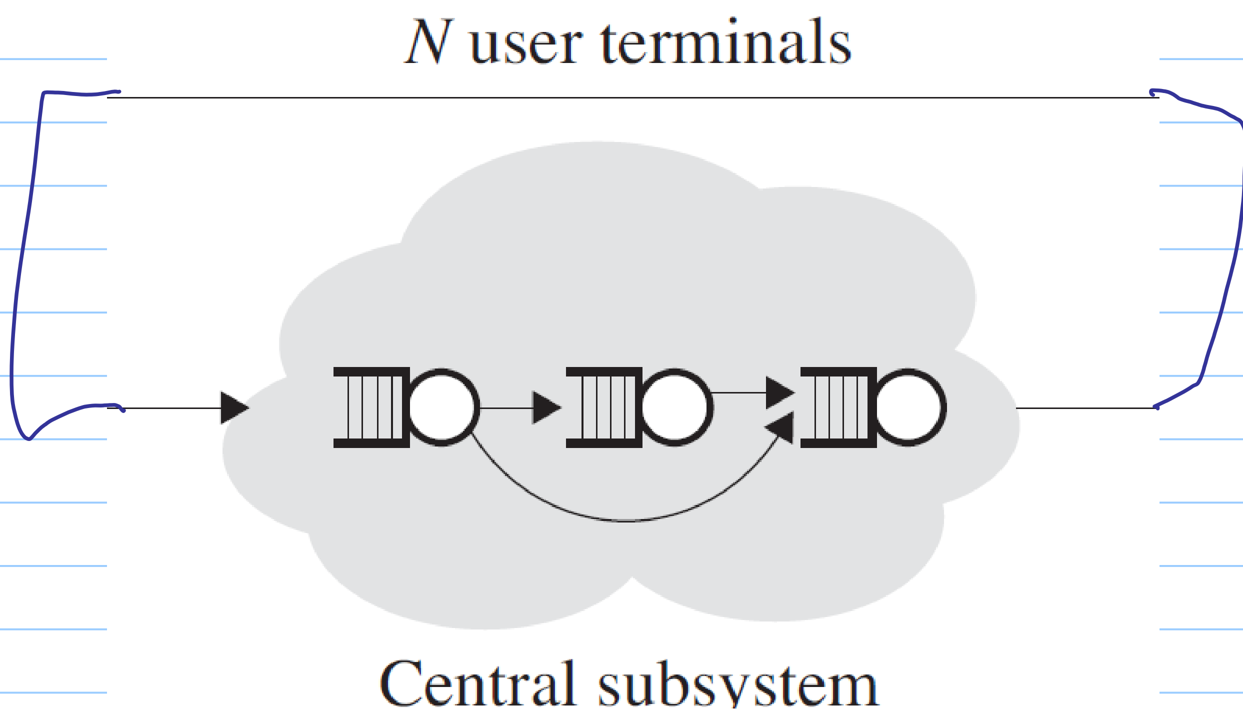
$$E[T] = E[R] + E[Z]$$

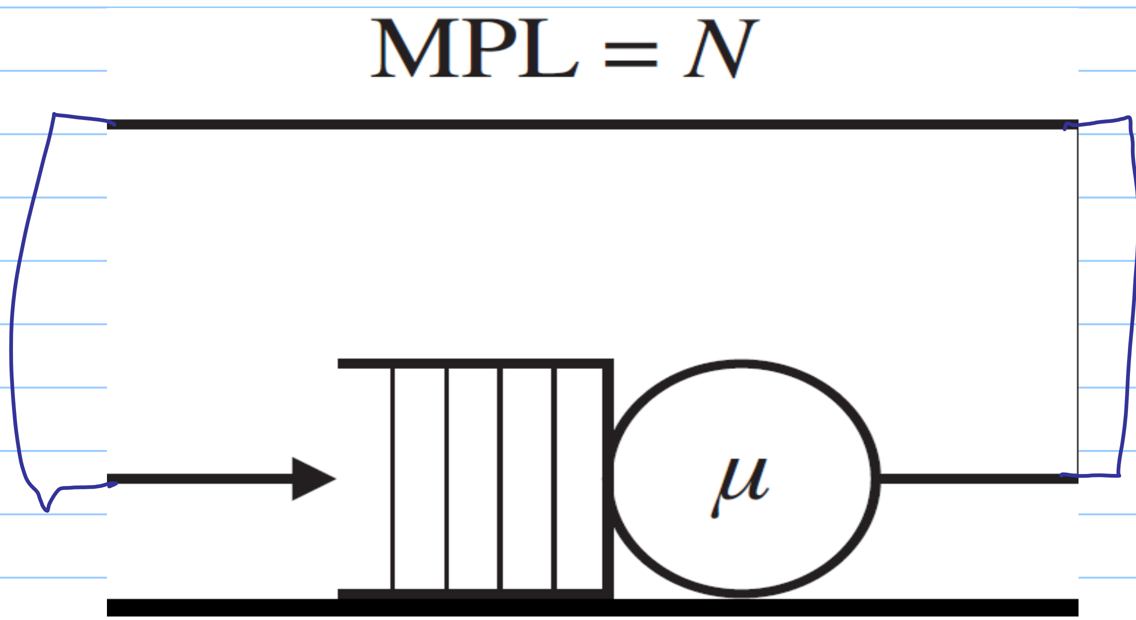
Time in system (for a job)

response time (in central subsystem)

Think time

Closed batch system

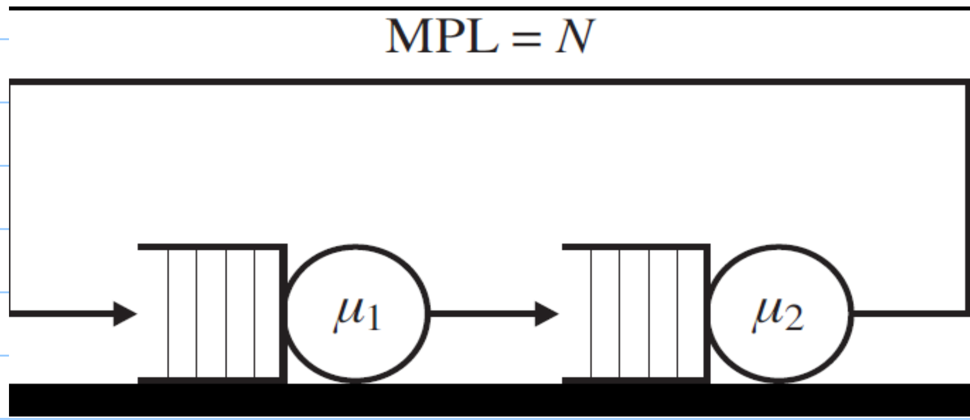




$X = \mu$

very different from the single server <u>open</u> system, where $X = \lambda$

Tandem closed batch system



What is the throughput?

HW 1: exercises 3.2, 3.4, 3.5, due in one week (1/27)