1 HW 3.5 - Intersection and DFA for 10th from right is 1 2008.xx.xx

1. Given $L_1$ and $L_2$ which are recognized by DFAs $M_1$ and $M_2$ construct a DFA that accepts $L_1 \cap L_2$.

Answer: Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$, and let $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$. Define $M_{INT} = (Q, \Sigma, \delta, q_0, F)$, where

- **States** $Q = Q_1 \times Q_2 = \{(x, y) \mid x \in Q_1 \text{andy} \in Q_2\}$.
- **Alphabet** Here we assume that $\Sigma$, the alphabet, is the same for all 3 DFAs.
- **Start state** $q_0 = (q_1, q_2)$
- **Final states** $F = F_1 \times F_2 = \{(x, y) \in Q \mid x \in F_1 \text{andy} \in F_2\}$.
- **Transition Function** For each element $a \in \Sigma$
  \[
  \delta((s_1, s_2), a) = \delta_1((s_1, a), \delta_2((s_2), a))
  \]

2. Using ten-tuples to represent states give the design of a DFA that recognizes the language

$L = \{x \in \{0, 1\}^* \mid \text{the tenth symbol from the right end of x is a } '1' \}$ To define the DFA we need only to specify all components:

- **States** $Q = \{(x_{10}, x_9, x_8 \cdots x_2, x_1) \mid x_i \in \{0, 1\}\}$ - note there are $2^{10}$ states.
- **Alphabet** $\Sigma = \{0, 1\}$
- **Start state** $q_0 = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
- **Final states** $F = \{(1, x_9, x_8 \cdots x_2, x_1) \mid x_i \in \{0, 1\}\}$ - note there are $2^9 = 512$ final or accepting states.
- **Transition Function** For each element $a \in \Sigma \delta(s, a)$ is defined by
  \[
  \delta((x_{10}, x_9, x_8 \cdots x_2, x_1), a) = (x_9, x_8, x_7 \cdots x_1, a)
  \]

The idea is to maintain in a list the last ten symbols and when the next character is considered shift everything one position and add the new character as the most recent symbol.