1 4.1.2 - Pumping Lemma

Prove \( L = \{ x \in \{0, 1\}^* \mid x = w w^R \} \) is not regular Using the Pumping Lemma

Proof:

- **Suppose** \( L \) is regular then by the pumping lemma there exists a DFA that recognizes \( L \) with \( n \) states.

- **Define the string** \( w = 0^n 110^n \)

- **Consider a partitioning of** \( w = xyz \) that satisfies
  i - \( |y| > 0 \) and
  ii - \( |xy| \leq n \) of the pumping lemma.

- **Find an i such that** \( xy^i z \notin L \)

  - Since \( |xy| \leq n \) this implies \( xy \) is all zeroes.

  - If we choose \( i=0 \) then \( xy^i z = xez = xz \) and
    \( xz \) has a pair of ones with fewer than \( n \) ones before them and \( n \) ones after them.

- Thus \( xz \notin L \). This contradiction shows that \( L \) is not regular.