1 Example Distinguished Set of Prefixes

In this example we show that \( L = \{0^n \mid n = k^2\} \) is not regular by building a set of \( n \) prefixes that are mutually distinguishable.

**Proof:** Let \( S = \{0^1, 0^2, \ldots, 0^n\} \). Then to show the elements of \( S \) are mutually distinguishable select two arbitrary elements say \( 0^i \) and \( 0^j \) and suppose without loss of generality that \( i < j \). Then

1. \( 0 < j - i < n \)
2. Let \( k \) be the smallest integer such that \( k > n \)
3. Then consider the string \( z = 0^{k^2 - i} \)
4. Since, \( 0^i z = 0^i 0^{k^2 - i} = 0^{k^2} \in L \)
5. But, now \( w = 0^j z = 0^j 0^{k^2 - i} = 0^{k^2} 0^{j - i} \)
6. \( |w| = k^2 + j - i < k^2 + n \) which is greater than \( k^2 \) but less than the next bigger perfect square \( (k + 1)^2 = k^2 + 2k + 1 \) since \( k \) was chosen to be larger than \( n \).
7. \( w = 0^j z \notin L \)
8. Thus each pair of prefixes in \( S \) is distinguishable by \( L \).
9. Thus \( L \) cannot be recognized by a DFA with \( n \) states.
10. Since \( n \), was arbitrary, \( L \) is not regular. Thus