csce750 — Analysis of Algorithms
Fall 2019 — Lecture Notes: Introduction

This document contains slides from the lecture, formatted to be suitable for printing or individual reading, and with some supplemental explanations added. It is intended as a supplement to, rather than a replacement for, the lectures themselves — you should not expect the notes to be self-contained or complete on their own.

1 What is an algorithm?

CLRS 1–2
2019-08-22

An *algorithm* is a sequence of unambiguous instructions for solving a problem, that is, for obtaining a required output for any legitimate input in a finite amount of time.

$$\text{instance} \longrightarrow \boxed{\text{algorithm}} \longrightarrow \text{output}$$

Analysis of algorithms is the quantitative study of the performance of algorithms, in terms of their run time, memory usage, or other properties.

2 What is this course about?

Most of the course will blend two parallel goals:

- **Techniques** for analyzing algorithms.
- **Applications** of those techniques to important algorithms and data structures.

3 Models of computation

We can make the idea of *sequence of instructions* precise by defining a *model of computation*.

One important early model of computation is the *Turing machine* which includes:

- A finite, non-empty set of **states** Q.
- A finine, non-empty set of **tape symbols** Γ.
- A **blank symbol** $b \in \Gamma$.
- A finite set of **input symbols** Σ.
- A **transition function** $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$,
- An **initial state** $q_0 \in Q$ and a set of **final states** $F \subseteq Q$.

> *Informally, we can think of a Turing machine as a finite state machine that reads and writes from an infinitely-long strip of tape. The main idea here is that, though the Turing machine model is very powerful and expressive, it is also cumbersome to use — essentially no one describes algorithms in it directly except in college classes on the theory of computation.*

4 *RAM model*

Another, more managable option:

Random-access machine (RAM) model (informal summary)

- Simple operations (arithmetic, comparison, conditional, *etc.*) each take the same, constant amount of time.
- Data stored in an infinite array of registers (0, 1, 2, ...), each of which can hold $c \log n$ bits.
  - n – problem size
  - c – some constant independent of n

> *In most cases, this level of detail is unnecessary for understanding how an algorithm works. However, it's important to have a formal model behind the scenes; without this, it's meaningless to try to prove anything about an algorithm or its performance.*

5 *Example: Sorting*

Sorting is a problem is that practically important and useful for illustrating many recurring ideas in algorithms.

- **Input:** A sequence of numbers $\langle a_1, \ldots, a_n \rangle$.
- **Output:** A reordering of those numbers, denoted $\langle a'_1, \ldots, a'_n \rangle$, such that

$$a'_1 \leq a'_2 \leq \cdots \leq a'_n.$$

> *Note that the idea of "sorting" is not restricted to just numbers. As long as the elements are drawn from a* totally ordered *set, then the problem is still well defined. We'll use numbers through this course because they make the intuition very easy.*

6 *Example: Insertion sort*

```
INSERTIONSORT(A)
  for j = 2, ..., A.length do
    k = A[j]
    i = j − 1
    while i > 0 and A[i] > k do
      A[i + 1] = A[i]
      i = i − 1
    end while
    A[i + 1] = k
  end for
```

7 *Example: Mergesort*

```
MERGESORT(A, ℓ, r)
  if ℓ < r then
    m = ⌊(ℓ + r)/2⌋
    MERGESORT(A, ℓ, m)
    MERGESORT(A, m + 1, r)
    MERGE(A, ℓ, m, r)        // CLRS pg 31
  end if
```