This optional assignment covers material from the lectures on Chapters 23, 24, 25, and 34, in preparation for the final exam.

Due: never

Not in book: Recall that Corollary 23.3 identifies certain edges that can safely be added to a partial minimum spanning tree. The diagram below shows a partially completed minimum spanning tree, with edges selected for the MST in bold. Which edges does this corollary guarantee are safe to add to the MST next?

![Graph Diagram](image)

Not in book: Use Kruskal’s algorithm to find a minimum spanning tree of the graph below. List the edges considered by the algorithm in order, and indicate whether each one is selected or rejected by the algorithm.

![Graph Diagram](image)

Page 637–638: Exercise 23.2-8

Not in book: Use Prim’s algorithm to find a minimum spanning tree of the graph below. Assume that all of the vertices except the root are added to the priority queue with key $\infty$ at the start. Use node $A$ as the root. List the vertices added to the MST, in order, along with any DECREASEKEY operations performed on the queue.

![Graph Diagram](image)
Not in book: Show, using a reduction from VERTEX COVER, that CLIQUE is NP-hard. (The CLIQUE problem is defined in Section 34.5.)

Not in book: Consider this problem:

**KARP-LETTERS:**

Instance: A list of strings of varying lengths, consisting of upper- and lower-case letters.

Question: Is there a way to select a letter from each string without choosing both versions of any letter?\(^1\)

Either prove that KARP-LETTERS $\in P$, or prove that KARP-LETTERS is NP-complete. (Hint: Only one of these options can be completed correctly.\(^2\)) For an NP-completeness proof, you may reduce from any problem identified as NP-hard in the lecture or in either textbook.

Page 1103: Problem 34-3

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\(^1\)For example, if the input strings are ‘Abc’, ‘BC’, ‘aB’, and ‘ac’, the correct answer is ‘Yes’, because we can choose ‘A’ from the first string, ‘B’ from the second, ‘B’ from the third, and ‘c’ from the fourth. If the strings are ‘AB’, ‘a’, and ‘b’, then the correct answer is ‘No’.

\(^2\)...unless P = NP.