
csce350 — Data Structures and Algorithms
Fall 2019 — Homework 14

Assigned: November 14

Due: November 21

Ruby is unsatisfied with the Fibonacci sequence discussed in class, because it is not defined for negative indices. To fix this, she subtracts $F(n - 1)$ from both sides of the original Fibonacci recurrence, to get

$$F(n - 2) = F(n) - F(n - 1).$$

From there, Ruby decides to change the indices (replacing n with $n + 2$), to get

$$\boxed{F(n) = F(n + 2) - F(n + 1)}.$$

She then notices that, for $n < 0$, this recurrence (along with the standard base cases $F(0) = F(1) = 1$), provides a well-defined sequence of Fibonacci-like numbers for negative integer indices. For example, $F(-1) = 0$, $F(-2) = 1$, and $F(-3) = -1$.

1. **Design** a dynamic programming algorithm that, given a negative number n , computes $F(n)$ by filling in an array of size $|n| + 2$. **Write** pseudocode for your algorithm.
2. **Optimize** your algorithm from part (a) by eliminating the need for the array. Instead use a small number of local variables. **Write** pseudocode for your algorithm.