CSCE274 Robotic Applications and Design
Fall 2022
Controllers

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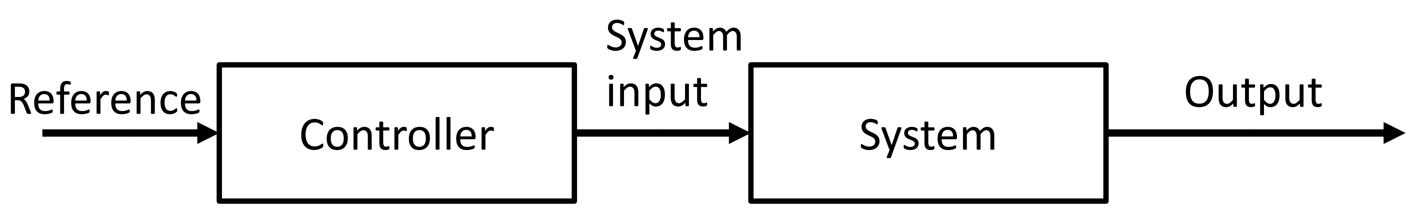
Control theory

- Control theory studies the behavior of systems whose behavior is governed by one or more inputs

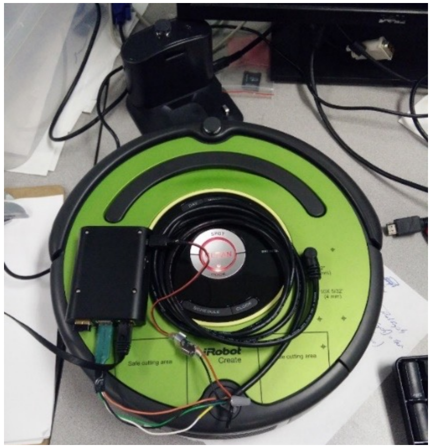
Open loop control

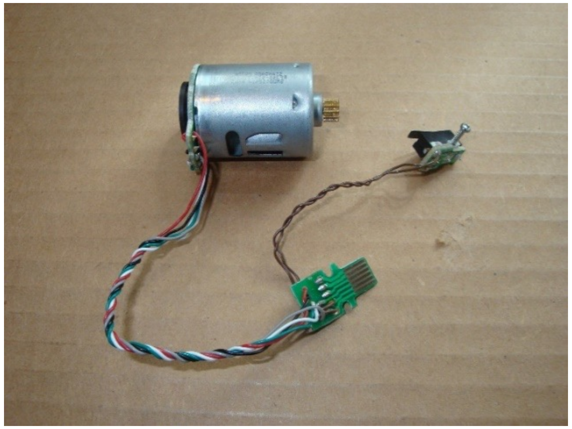
- *Open-loop controller* or *non-feedback controller* is a type of controller that computes the system input only using the current state and its model of the system



Open loop control

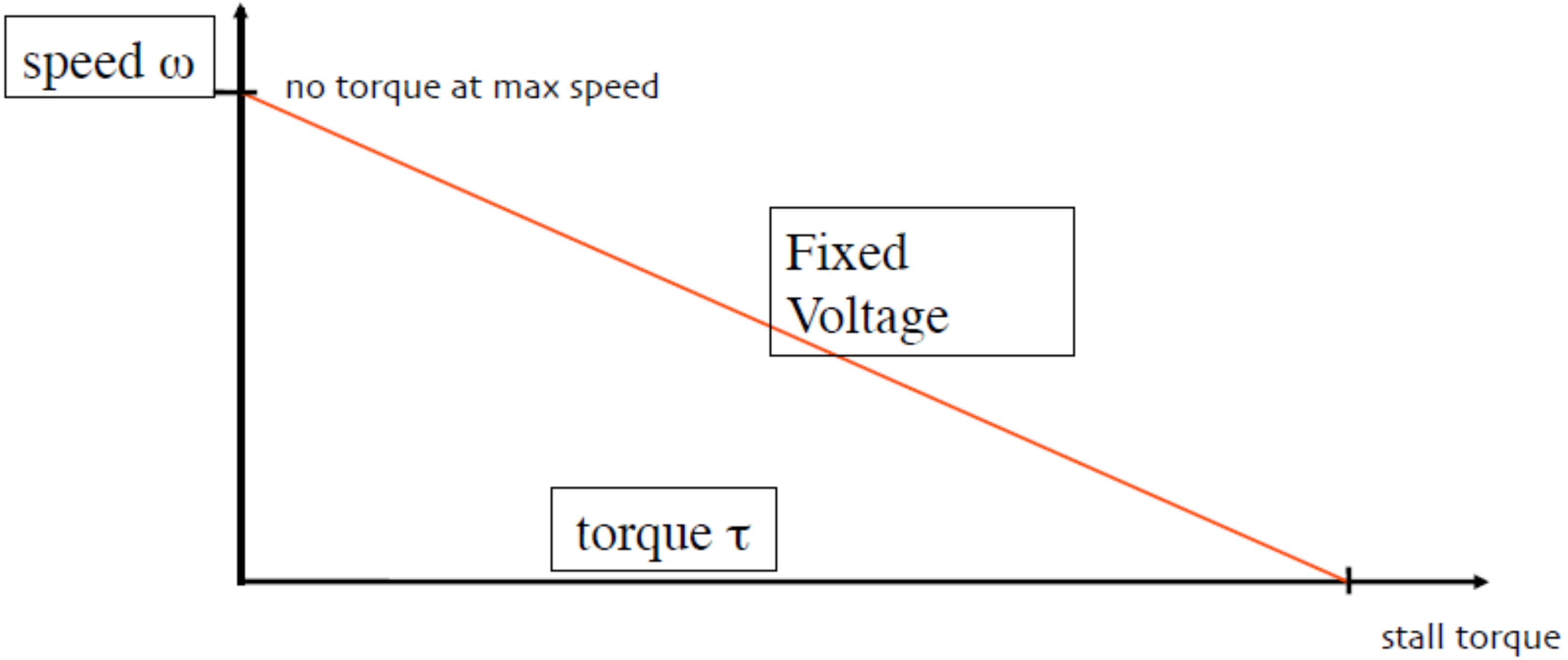
- Example:
  - Move a differential drive robot by spinning motors at a given angular velocity
  - Apply fixed voltage to it and never check to see if it is rotating properly





Open loop control

- Example:
  - Changing load on the motor changes also the output velocity



Open loop control

- Example:
  - Target reaching

Source: konozlearning.com

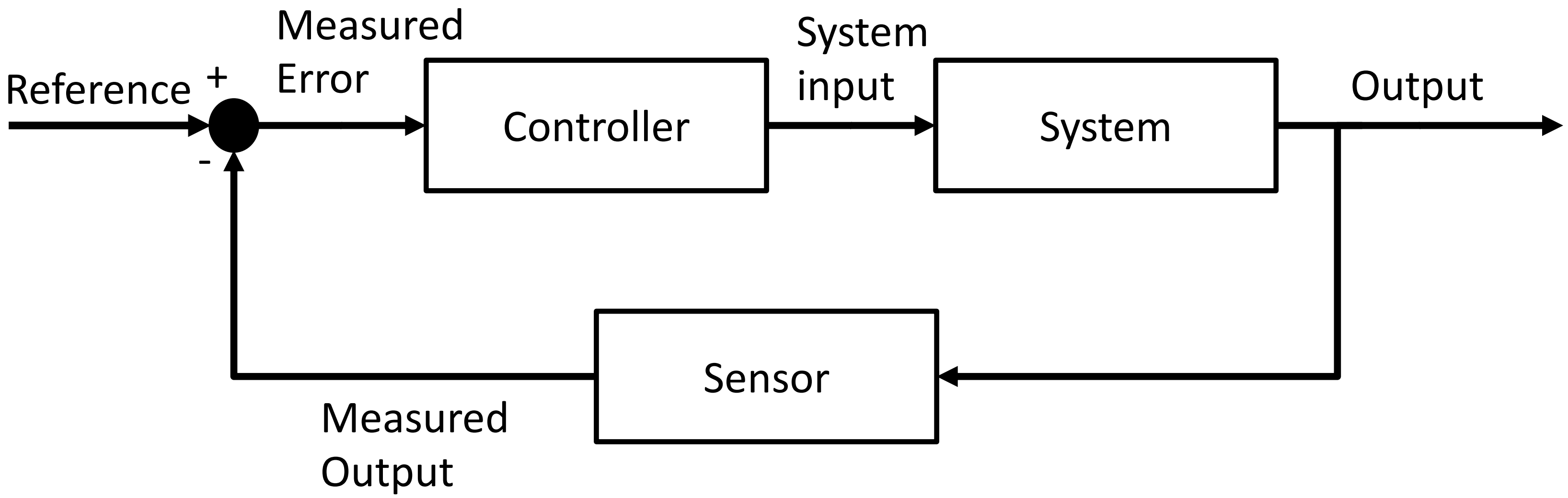
Open loop control

- Example:
  - Target reaching

Source: konozlearning.com

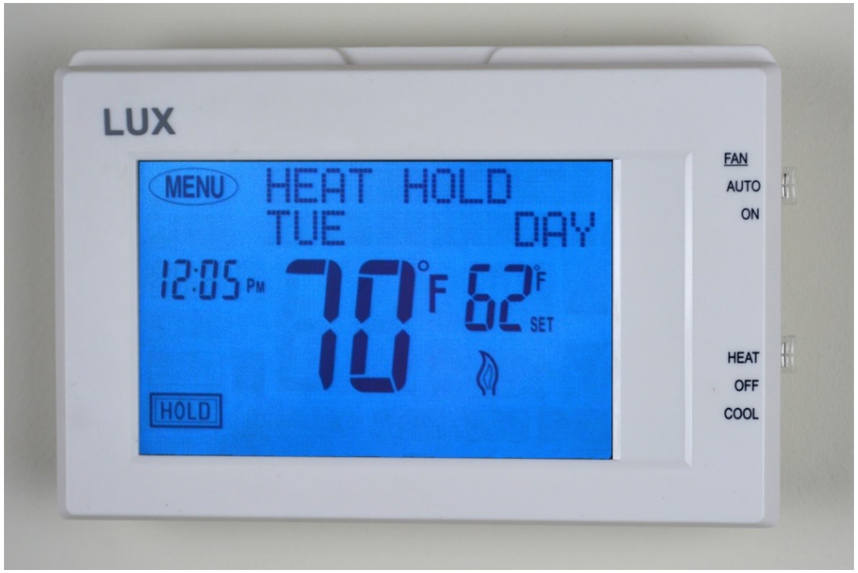
Closed loop control

- *Closed loop control* or *feedback control* is a type of controller that computes the system input using also a sensor to measure the error that is taken into account



Closed loop control

- Example:
  – Thermostat



Thermostat – Source: wikipedia.com

Closed loop control

- The goal of a feedback controller is to achieve and maintain a desired state (*set point*) by using the information coming from the sensor(s)

- *Error* is the difference between the current and desired state

- *Sampling rate* is the frequency at which the sensors read new information that can be used to compute the error

Notation

- $X(t)$ *state space*
- $x(t)$ *state* at time t
- U *input space* (also called *action space* or *control space*)
- $u(t)$ *input* (also called *action* or *control*) at time t
- $e(t)$ *error*

Basic controller functions

- Proportional term accounts for present errors

$$u(t) = K_p e(t) \qquad \frac{U(s)}{E(s)} = K_p$$

where

– K_p is a constant that is called *proportional gain*

Basic controller functions

- Integral term accounts for past errors

$$u(t) = K_i \int_0^t e(t)dt \qquad \frac{U(s)}{E(s)} = \frac{K_i}{s}$$

where

– K_i is a constant that is called *integral gain*

Basic controller functions

- Derivative term accounts for future errors

$$u(t) = K_d \frac{d}{dt} e(t) \qquad \frac{U(s)}{E(s)} = K_d s$$

where

– K_d is a constant that is called *derivative gain*

Basic controller functions

- It is possible to combine the different feedback controllers
  - PD controller

$$u(t) = K_p e(t) + K_d \frac{d}{dt} e(t)$$

  - PID controller

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{d}{dt} e(t)$$

How to tune the gains?

- The usual case is to tune them experimentally
  - Different methods to do it

- When tuning, it is fundamental to consider
  - Stability
  - Oscillation
  - Response time
  - Steady-state error: final difference between the state and the set point

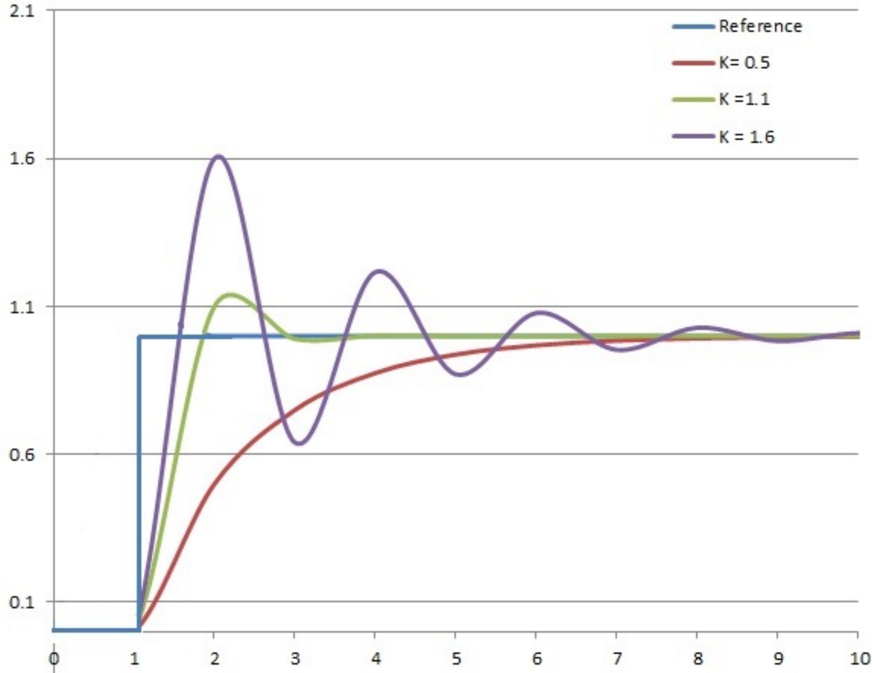
Closed loop control

- Example:
  - Target reaching

Source: konozlearning.com

PID examples

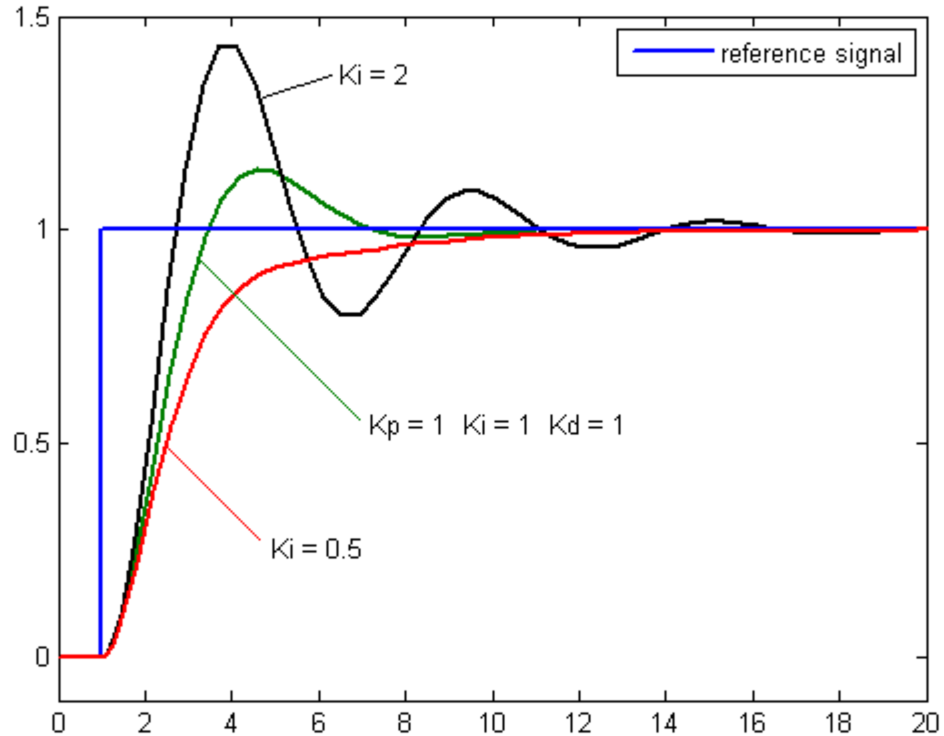
- Changes to K_p
  - High proportional gain will increase the speed of the control system response: possibly unstable system
  - Small gain results in small output response to a large input error: possibly less responsive controller



Source:Wikipedia.org

PID examples

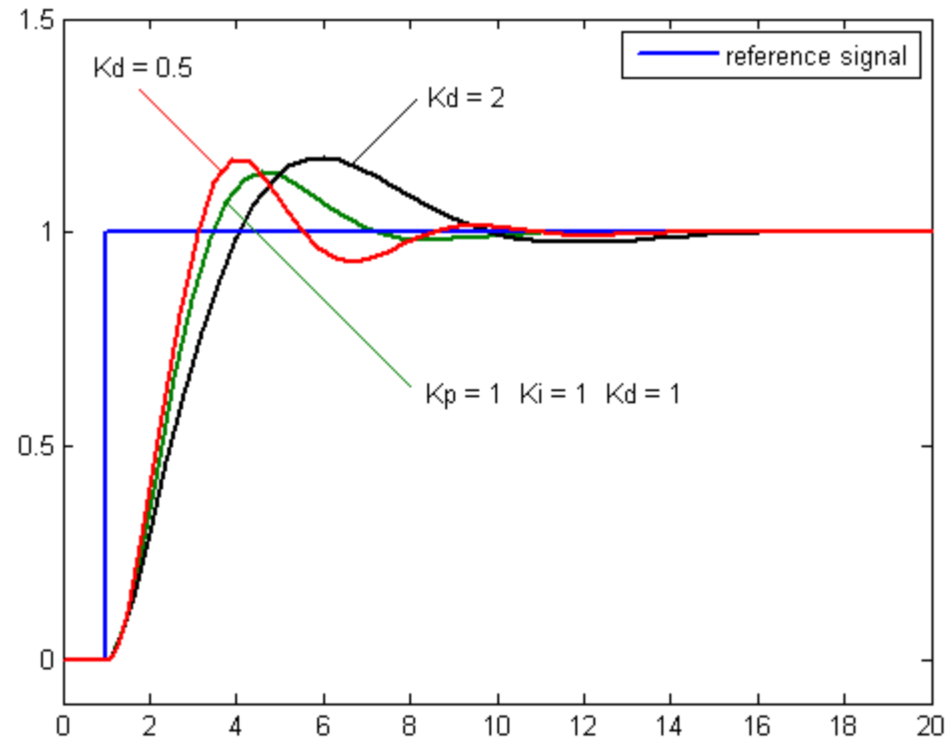
- Changes to K_i
  - High integral gain can result in overshooting the setpoint value
  - Small gain can result in slower rise time



Source:Wikipedia.org

PID examples

- Changes to K_d
  - High derivative gain increases the speed of the overall control system response
  - Small gain can result in slower rise time



Source:Wikipedia.org

Discrete time controller

- Systems are usually discrete time, thus discrete approximations are needed

$$u_k = K_p e_k + K_i \Delta t \sum_{i=1}^{k} e_i + K_d \frac{e_k - e_{k-1}}{\Delta t}$$

Discrete time controller – Exercise

- Exercise: given the following
  - Set point 200
  - $K_p = K_i = K_d = 2$
  - Sampling rate (Δt) is 1 second
  - $x_1 = 205$
  - $x_2 = 204$
  - $x_3 = 198$

  Compute the output of the controller after the third sensor reading

Discrete time controller – Exercise

- Errors
  - $e_1 = 200 - 205 = -5$
  - $e_2 = 200 - 204 = -4$
  - $e_3 = 200 - 198 = 2$

Discrete time controller – Exercise

- Errors
  - $e_1 = 205 - 200 = 5$
  - $e_2 = 204 - 200 = 4$
  - $e_3 = 198 - 200 = -2$

Discrete time controller – Exercise

- Controls over time, applying the discrete PID controller

$$u_3 = K_p e_3 + K_i \Delta t \sum_{i=1}^{3} e_i + K_d \frac{e_3 - e_2}{\Delta t}$$

$$u_3 = 2(-2) + 2(1(5 + 4 + (-2))) + 2\frac{-2-4}{1}$$

$$u_3 = -2$$