The extended group consensus based evidential reasoning approach for multiple attributive group decision analysis problems with missing assessments using a feedback mechanism

Chao FU[a,b,c,*], Michael Huhns[c], Shanlin Yang[a,b]

[a]*School of Management, Hefei University of Technology, Hefei, Box 270, Hefei 230009, Anhui, P.R. China*

[b]*Key Laboratory of Process Optimization and Intelligent Decision-making, Ministry of Education, Hefei 230009, Anhui, P.R. China*

[c]*Department of Computer Science and Engineering, University of South Carolina, Columbia 29208, South Carolina, USA*

Abstract

Many multiple attributive group decision analysis (MAGDA) problems under the uncertain environment with group consensus (GC) requirements can be modeled by a GC based evidential reasoning (GCER) approach. However, due to the limitation of knowledge, experience and provided data, experts may feel difficult to give opinions on specific attributes for specific or all alternatives. Additionally, group analysis and discussion with no guidance may not be the best way to speed up the convergence of GC reaching process. In this paper, the GCER approach is extended to deal with MAGDA problems with missing assessments and GC requirements. Experts decide their assessments on missing attributes based on the recommendations generated by solving optimization problems constructed according to other experts' assessments to reach the maximal GC on the specific attribute of specific alternative. Further, a feedback mechanism is introduced to help quickly reach predefined GC requirements. Aiming at three GC levels, the attribute, alternative and global level, the corresponding set of identification rules are investigated to indicate that specific experts are recommended to renew assessments on specific attributes for specific alternatives. After the identification, a set of suggestion rules are presented to generate appropriate recommendations for experts to renew assessments. An engineering project management software selection problem is solved by the extended GCER approach to demonstrate its detailed implementation process, and its validity and applicability.

---

* Corresponding author. Tel: +86 0551 2904930; fax: +86 0551 2905263.

E-mail address: wls_fuchao@163.com (C. Fu).

Keywords: Decision analysis; Multiple attributive group decision analysis; Evidential reasoning approach; Group consensus; Missing assessments; Feedback mechanism

1. Introduction

An evidential reasoning (ER) approach developed in the 1990s and in recent years (Yang and Sen, 1994,1997; Yang and Singh, 1994; Yang, 2001; Yang and Xu, 2002a,b; Wang et al., 2006b; Yang et al., 2006; Guo et al., 2007; Chin et al., 2009) can model multiple attribute decision analysis (MADA) problems with both quantitative and qualitative attributes under the uncertain environment. Furthermore, the extensions of ER approach (Chin et al., 2009, 2008) can model multiple attributive group decision analysis (MAGDA) problems under the uncertain environment. To adequately and properly take into account the preferences of all experts, an extension of ER approach called group consensus based ER (GCER) approach (Fu and Yang, 2010) is proposed to generate a common solution for MAGDA problems with group consensus (GC) requirements.

In the GCER approach, consensus measures are constructed on either of attributes and alternatives and correspondingly the GC on both attributes and alternatives can be checked. Furthermore, many important factors including experts' utilities, the subjective weights of experts, the flexibility in consensus measures, the special design for MAGDA problems are also considered in the GCER approach. Therefore, it is more applicable than other existing GC related group decision analysis (GDA) approaches (e.g., Bordogna et al., 1997; Herrera et al., 1996, 1997; Ben-Arieh and Chen, 2006; Dong et al., 2009; Herrera-Viedma et al., 2005; Dong et al., 2008; Mata et al., 2009; Cabrerizo et al., 2009; Szmidt and Kacprzyk, 2003; Herrera-Viedma et al., 2007, 2004; Choudhury et al., 2006; Alonso et al., 2008) in real cases.

However, due to the limitation of knowledge, experience and provided data about the problem domain (Kim and Ahn, 1999; Kim and Han, 1999; Kim et al., 1999; Herrera-Viedma et al., 2007a, 2007b), experts may feel difficult to give appropriate assessments on specific attributes (also called missing attributes) for specific or all alternatives. This situation cannot be dealt with by the GCER approach. In addition, the specified rounds of group analysis and discussion (GAD) with no guidance may spend a long period of time and probably have no GC based solution.

In this paper, the GCER approach is extended to deal with the situation of missing attributes and speed up the convergence of GC reaching process. By solving optimization problems constructed based on the idea of reaching the maximal GC on missing attributes when at least one expert's assessment is known, the recommendations on missing attributes can be generated. Experts decide their assessments on missing attributes based on the recommendations according to their preferences after the GAD, in which they can know the reasons of others' assessments. Further, a feedback mechanism, which involves a set of identification rules at three different levels including the attribute, alternative and global level, and a set of suggestion rules, is designed to speed up the convergence of GC reaching process to quickly find a GC based solution. The set of identification rules at a specific level can generate a set in which specific experts are recommended to renew assessments on specific attributes for specific alternatives. And the set of suggestion rules give recommendations to specific experts in the set about how to renew their assessments.

The rest of this paper is organized as follows. Section 2 gives the preliminaries related to the extended GCER approach. Section 3 interprets the extended GCER approach in detail. An engineering project management software selection problem is solved in Section 4 to demonstrate a detailed implementation process of the extended GCER approach, and its validity and applicability. Section 5 compares the extended GCER approach with other GC based approaches which can deal with the situation of missing attributes and provide the feedback to experts to accelerate the convergence of GC reaching process. Finally, this paper is concluded in Section 6.

2. Preliminaries

2.1. Basics of Dempster-Shafer theory

Dempster-Shafer theory (DST) was first proposed by Dempster in the 1960s and mathematically developed by Shafer in the 1970s (Dempster, 1967; Shafer, 1976). It provides a distributed framework to model probabilistic uncertainties.

Let $\Omega=\{H_1,H_2,\ldots,H_N\}$ be a collectively exhaustive and mutually exclusive set of propositions, referred to as a frame of discernment. A belief structure (BS) m is defined as a mapping from 2^{Ω} to 1 verifying $\sum_{A\subseteq\Omega} m(A)=1$ (Denoeux, 1999). Each subset A of Ω such that $m(A)>0$ is called a focal element of m. All focal elements of m are denoted by $\Theta(m)$.

Given two BSs m_1 and m_2 on Ω derived from two reliable distinct sources, the combined BS with Dempster's rule of combination from m_1 and m_2 is defined as $m_{12}=m_1\oplus m_2$, which is

$$m_{12}(A)=\frac{\sum_{B,C\subseteq\Omega,B\cap C=A} m_1(B)m_2(C)}{1-\sum_{B,C\subseteq\Omega,B\cap C=\varnothing} m_1(B)m_2(C)},\quad \forall A\subseteq\Omega,\quad A\neq\varnothing,$$

when $\sum_{B,C\subseteq\Omega,B\cap C=\varnothing} m_1(B)m_2(C)\neq 1$.

Here, $\sum_{B,C\subseteq\Omega,B\cap C=\varnothing} m_1(B)m_2(C)$ is the mass of the combined belief assigned to the empty set before the normalization. In the following, we denote it by $m_{\oplus}(\varnothing)(m_1,m_2)$. Dempster's rule is meaningful and applicable only when $m_{\oplus}(\varnothing)(m_1,m_2)\neq 1$ holds.

Let m be a BS on Ω. Its associated pignistic probability function $BetP(m)$ in the transferred belief model is defined as (Smets, 2004; Smets and Kennes, 1994)

$$BetP(m)(w)=\sum_{A\subseteq\Omega,w\in A}\frac{1}{|A|}\frac{m(A)}{1-m(\varnothing)},\quad m(\varnothing)\neq 1,$$

where $|A|$ is the cardinality of subset A.

$BetP(m)$ can be extended as a function on 2^{Ω} as (Shafer, 1976)

$$BetP(m)(A)=\sum_{B\subseteq\Omega}\frac{|A\cap B|}{|A|}\frac{m(B)}{1-m(\varnothing)},\quad \forall A\subseteq\Omega.$$

The transformation from m to $BetP(m)$ is named as the pignistic transformation.

2.2. The GCER distributed modeling framework for MAGDA problems

Suppose a MAGDA problem includes T experts t_j (j=1,…,T) and a manager. The relative weights of T experts on the attribute e_i for the alternative a_l are denoted by $\lambda(e_i(a_l))=(\lambda^1(e_i(a_l)), \lambda^2(e_i(a_l)), \ldots, \lambda^T(e_i(a_l)))$ such that

$$0\le\lambda^j(e_i(a_l))\le 1 \text{ and } \sum_{j=1}^{T}\lambda^j(e_i(a_l))=1. \quad (1)$$

All experts deal with a common MADA problem which has M alternatives a_l (l=1,…,M), on the upper level attribute, referred to as a general attribute, and L lower level attributes e_i (i=1,…,L), called basic attributes. The relative weights of L basic attributes are denoted by $w=(w_1,w_2,\ldots,w_L)$ such that

$$0\le w_i\le 1 \text{ and } \sum_{i=1}^{L}w_i=1. \quad (2)$$

Suppose H_n (n=1,…,N) denotes a set of grades. M alternatives are assessed at L attributes using H_n (n=1,…,N). Let $B(e_i(a_l))=\{(H_n, \beta_{n,i}(a_l)), n=1,\ldots,N\}$ denote the distributed assessment vector on the attribute e_i for the alternative a_l to the grade H_n with the belief degree of $\beta_{n,i}(a_l)$ such that $\beta_{n,i}(a_l)\ge 0$, $\sum_{n=1}^{N}\beta_{n,i}(a_l)\le 1$, and $\sum_{n=1}^{N}\beta_{n,i}(a_l)+\beta_{\Omega,i}(a_l)=1$. If $\beta_{\Omega,i}(a_l)=0$, then the assessment is complete; otherwise, it is incomplete.

After M alternatives are all assessed on L basic attributes, a belief decision matrix will be achieved, which is

$$S_g=B(e_i(a_l))_{L\times M}. \quad (3)$$

Aided by the principle of utility equivalence (Yang, 2001), quantitative attributes can also be modeled by using the defined assessment grades (Wang et al., 2006b).

The expert t_j gives a belief decision matrix $S_g^j=B^j(e_i(a_l))_{L\times M}$, where $B^j(e_i(a_l))=\{(H_n, \beta_{n,i}^j(a_l)), n=1,\ldots,N\}$ such that $\sum_{n=1}^{N}\beta_{n,i}^j(a_l)+\beta_{\Omega,i}^j(a_l)=1$.

2.3. The GC in the GCER approach

In the GCER approach, the GC is constructed at three levels which are the attribute, alternative and global level, which is defined as

$$gc(e_i(a_l))=\frac{\sum_{j=1}^{T}\frac{\sum_{k=1,k\ne j}^{T}cm(\bar{V}^j(e_i(a_l)),\bar{V}^k(e_i(a_l)))}{T-1}}{T}, \quad (4)$$

$gc(a_l)=\sum_{i=1}^{L} w_i gc(e_i(a_l))$, (5)

and $ggc=\sum_{l=1}^{M} gc(a_l)/M$, (6)

respectively. Here, cm and $\bar{V}^j(e_i(a_l))$ (j=1,…,T) denote the compatibility measure between two BSs and the unified assessment with no utility difference from the expert t_j on the attribute e_i for the alternative a_l, respectively.

Given a threshold vector $\delta=(\delta_1,\ldots,\delta_L)$ such that $0\le\delta_i\le1$ (i=1,…,L), two thresholds δ_M and δ_G such that $0\le\delta_M\le1$ and $0\le\delta_G\le1$, respectively, the GC at three levels can be checked as

$gc(e_i(a_l))\ge\delta_i$, i=1,…,L, l=1,…,M, (7)

$gc(a_l)\ge\delta_M$, l=1,…,M, (8)

and $ggc\ge\delta_G$, (9)

respectively.

The details about the GC can be found in Sections A.1 and A.2 in Appendix A and (Fu and Yang, 2010).

3. The extended GCER approach

In the GCER approach, all experts participating in the GDA process can give their assessments on each attribute for each alternative. However, in some cases some experts may feel difficult to give appropriate assessments on missing attributes for specific or all alternatives, due to the limitation of knowledge, experience and provided data. In the extended GCER approach, some recommendations on missing attributes will be generated based on the idea of reaching the maximal GC on missing attributes when at least one expert's assessment is known, and given to experts to form their assessments after the GAD. This recommendation strategy can undoubtedly speed up the convergence of GC reaching process.

After that, a feedback mechanism including a set of identification rules at three levels and a set of suggestion rules is introduced to help quickly reach the predefined GC.

3.1. The recommendations on missing attributes

For specific experts who cannot give effective assessments on missing attributes, the extended GCER approach generates recommendations by reaching the maximum of GC on missing attributes when at least one expert express the opinion on missing attributes. Then all experts have a GAD to freely express opinions and study from each other. After that the specific experts give assessments based on the recommendations according to their preferences and the extent to which they agree with other experts who give assessments. There is no doubt that this strategy of generating recommendations can speed up the convergence of GC reaching process.

For any missing attribute e_{si} for a specific alternative a_{sl}, suppose m ($1 \le m \le T$) experts give assessments and $T-m$ experts cannot give assessments. To reach the maximal GC on the missing attribute e_{si} for the specific alternative a_{sl}, the recommendations to $T-m$ experts can be obtained by solving the following optimization problem.

MAX $gc(e_{si}(a_{sl}))$

s.t. $V^j(e_{si}(a_{sl}))$, $j=1,\ldots,m$,

$V^k(e_{si}(a_{sl}))$, $k=m+1,\ldots,T$.

Here, $V^j(e_{si}(a_{sl}))=B^j(e_i(a_l))+\beta^j_{\Omega,i}(a_l)$ and $V^k(e_{si}(a_{sl}))=B^k(e_i(a_l))+\beta^k_{\Omega,i}(a_l)$ denote the assessments of experts t_j ($j=1,\ldots,m$) and t_k ($j=\mathrm{m}+1,\ldots,T$) on their own utilities, respectively, so that the recommendation $V^k(e_{si}(a_{sl}))$ ($k=m+1,\ldots,T$) can be used for the expert t_j to give the assessment on the missing attribute e_{si} for the alternative a_{sl}. In the construction of the above optimization problem, $V^j(e_{si}(a_{sl}))$ and $V^k(e_{si}(a_{sl}))$ are transformed to $\bar{V}^j(e_i(a_l))$ and $\bar{V}^k(e_i(a_l))$, the unified assessments with no utility difference, respectively.

With lingo software package, the above problem can be solved to generate recommendations.

3.2. The feedback mechanism

A feedback mechanism is developed to decrease the rounds of GAD and help quickly reach the predefined GC. It includes a set of identification rules at three levels and a set of suggestion rules. The set of identification rules at a specific level are used to indicate specific experts whose assessments on specific attributes for specific alternatives are damaging to the GC at the specific level. And the set of suggestion rules generate recommendations to specific experts about how to renew their assessments.

3.2.1. The proximity measure

Except for the GC, a proximity measure (PM) is needed to effectively construct the set of identification rules to deal with different GC at three levels, similar to Mata et al.'s adaptive consensus model (Mata et al., 2009). It measures the compatibility between the individual opinions and the group opinion and similarly will be constructed at three levels.

The PM at the attribute level is defined as

$$pm^j(e_i(a_l))=cm(\overline{V}^j(e_i(a_l)), \overline{B}(e_i(a_l))),\ j=1,\ldots,T. \tag{10}$$

Corresponding to the GC at the alternative and global levels, the PMs at the alternative and global levels can be defined as

$$pm^j(a_l)=\sum\nolimits_{i=1}^{L} w_i \cdot pm^j(e_i(a_l)), \tag{11}$$

and $$pm^j=\sum\nolimits_{l=1}^{M} pm(a_l)/M\ . \tag{12}$$

3.2.2. The identification rules

In the GC reaching process, the GC has been gradually increased from a relative low value at the beginning to the required value at last. Based on Mata et al.'s idea (Mata, 2009), the GC is divided into three states which are very low, low and medium. At different states, there is different number of experts' assessments damaging to the GC. Therefore, different strategies are correspondingly needed to identify specific experts who should renew their assessments on specific attributes for specific alternatives.

Corresponding to the GC requirements at three levels, the set of identification rules are constructed at three levels, respectively.

(1) The identification rule at the attribute level.

When the GC requirement at the attribute level is not reached, i.e. $gc(e_i(a_l))<\delta_i$, the GC can only be divided into two states which are very low and medium since only $pm^j(e_i(a_l))$ can be used to identify the assessments damaging to the GC. On condition that the GC is very low, all experts instead of a part of them will be recommended to renew their assessments to avoid the GC reaching process is guided by some experts imposing assessments. After several rounds of GAD and assessments renewing, the GC can be increasingly at a medium state. In this situation, a part of experts rather than all experts should be advised to modify their assessments. Given the threshold ρ_1, the GC can be decided at the very low state when $gc(e_i(a_l))\leq\rho_1$; and it can be decided at the medium state when $\rho_1<gc(e_i(a_l))<\delta_i$.

(A) The GC is very low

When the GC is very low, the set of assessments to be renewed can be identified as $ASSATT^{VL}=\{gc(e_i(a_l))\leq\rho_1\}$. All experts are advised to renew assessments on each element in $ASSATT^{VL}$.

(B) The GC is medium

When the GC is increased to be medium, specific experts can be identified to change assessments by using the PM at the attribute level given a threshold. The value of threshold may be static and fixed before the beginning of the GC reaching process or dynamic associated with the proximity values after each round of GAD and assessments renewing. Based on Mata et al.'s idea (Mata, 2009), a dynamic threshold is considered more reasonable and one of its possible value which is $\sum_{j=1}^{T} pm^j(e_i(a_l))\big/T$ is selected.

As a result, the set of assessments to be renewed can be identified as $ASSATT_j^M=\{\rho_1<gc(e_i(a_l))<\delta_i,\ pm^j(e_i(a_l))<\sum_{j=1}^{T} pm^j(e_i(a_l))\big/T\}$. Each expert t_j in $ASSATT_j^M$ will renew assessments on each element in $ASSATT_j^M$.

(2) The identification rule at the alternative level.

The GC at the alternative level can be in three states which are very low, low and medium due to the application of the PMs at the alternative and attribute levels to identify the assessments damaging to the GC. Given the thresholds ρ_1 and ρ_2, the GC can be decided at the very low, low and medium states when $gc(a_l)\leq\rho_1$, $\rho_1<gc(a_l)\leq\rho_2$ and $\rho_2<gc(a_l)<\delta_M$, respectively.

On the other hand, the thresholds constraining the GC at the attribute level and the

PMs at the alternative and attribute levels are dynamic rather than static similar to the identification rule at the attribute level.

(A) The GC is very low

At the beginning of GC reaching process when the GC is very low, the set of assessments to be renewed can be identified as $ASSALT^{VL}$={$gc(a_l)\leq\rho_1$, $gc(e_i(a_l))< \sum_{l=1}^{M}\sum_{i=1}^{L} gc(e_i(a_l)) / L\times M$ }. All experts are recommended to renew assessments on each element in $ASSALT^{VL}$.

(B) The GC is low

Along with the increase of GC from being very low to being low, the set of assessments to be renewed is changed as $ASSALT_j^L$ ={$\rho_1<gc(a_l)\leq\rho_2$, $gc(e_i(a_l))< \sum_{l=1}^{M}\sum_{i=1}^{L} gc(e_i(a_l)) / L\times M$, $pm^j(e_i(a_l))< \sum_{j=1}^{T} pm^j(e_i(a_l)) / T$ }. Each expert t_j in $ASSALT_j^L$ will renew assessments on each element in $ASSALT_j^L$.

(C) The GC is medium

When the GC continues to be increased to be in the interval of ρ_2 and δ_M, the set of assessments to be renewed is further changed as $ASSALT_j^M$ ={$\rho_2<gc(a_l)<\delta_M$, $gc(e_i(a_l))< \sum_{l=1}^{M}\sum_{i=1}^{L} gc(e_i(a_l)) / L\times M$, $pm^j(e_i(a_l))< \sum_{j=1}^{T} pm^j(e_i(a_l)) / T$, $pm^j(a_l)<\sum_{j=1}^{T} pm^j(a_l) / T$, }. Each expert t_j in $ASSALT_j^M$ will renew assessments on each element in $ASSALT_j^M$.

(3) The identification rule at the global level.

Similar to the identification rule at the alternative level, the one at the global level can be constructed and introduced in the following.

(A) The GC is very low

When the GC is very low, the set of assessments to be renewed can be identified as $ASSGLO^{VL}$={$ggc\leq\rho_1$, $gc(e_i(a_l))< \sum_{l=1}^{M}\sum_{i=1}^{L} gc(e_i(a_l)) / L\times M$, $pm^j(e_i(a_l))< \sum_{j=1}^{T} pm^j(e_i(a_l)) / T$ }. All experts are recommended to renew assessments on each element in $ASSGLO^{VL}$.

(B) The GC is low

If the GC is increased to be low, the set of assessments to be renewed is changed as $ASSGLO_j^L$ ={ρ_1<ggc≤ρ_2, $gc(e_i(a_l))$< $\sum_{l=1}^{M}\sum_{i=1}^{L} gc(e_i(a_l))\big/L\times M$, $gc(a_l)$<$\sum_{l=1}^{M} gc(a_l)\big/M$, $pm^j(e_i(a_l))$<$\sum_{j=1}^{T} pm^j(e_i(a_l))\big/T$, $pm^j(a_l)$<$\sum_{j=1}^{T} pm^j(a_l)\big/T$ }. Each expert t_j in $ASSGLO_j^L$ will renew assessments on each element in $ASSGLO_j^L$.

(C) The GC is medium

When the GC continues to be increased to be medium, the set of assessments to be renewed is further changed as $ASSGLO_j^M$ ={ρ_2<ggc<δ_G, $gc(e_i(a_l))$< $\sum_{l=1}^{M}\sum_{i=1}^{L} gc(e_i(a_l))\big/L\times M$, $gc(a_l)$< $\sum_{l=1}^{M} gc(a_l)\big/M$, $pm^j(e_i(a_l))$< $\sum_{j=1}^{T} pm^j(e_i(a_l))\big/T$, $pm^j(a_l)$< $\sum_{j=1}^{T} pm^j(a_l)\big/T$, pm^j< $\sum_{j=1}^{T} pm^j\big/T$ }. Each expert t_j in $ASSGLO_j^M$ will renew assessments on each element in $ASSGLO_j^M$.

3.2.3. The suggestion rules

After the assessments damaging to the GC have been identified, a set of suggestion rules are defined in the feedback mechanism to guide experts to renew their assessments.

Suppose MV={(H_n, 0), n=1,…,N−1, (H_N, 1)}.

(a) The suggestion rule 1. If $cm(\bar{V}^j(e_i(a_l))$, MV)<$cm(\bar{V}(e_i(a_l))$, MV), the expert t_j will be recommended to increase the assessment $\bar{V}^j(e_i(a_l))$.

(b) The suggestion rule 2. If $cm(\bar{V}^j(e_i(a_l))$, MV)>$cm(\bar{V}(e_i(a_l))$, MV), the expert t_j will be recommended to decrease the assessment $\bar{V}^j(e_i(a_l))$.

(c) The suggestion rule 3. If $cm(\bar{V}^j(e_i(a_l))$, MV)=$cm(\bar{V}(e_i(a_l))$, MV), the expert t_j will be recommended not to change the assessment $\bar{V}^j(e_i(a_l))$.

3.3. The procedure of extended GCER approach

The procedure of extended GCER approach is shown in Figure 1, which will be elaborated step by step.

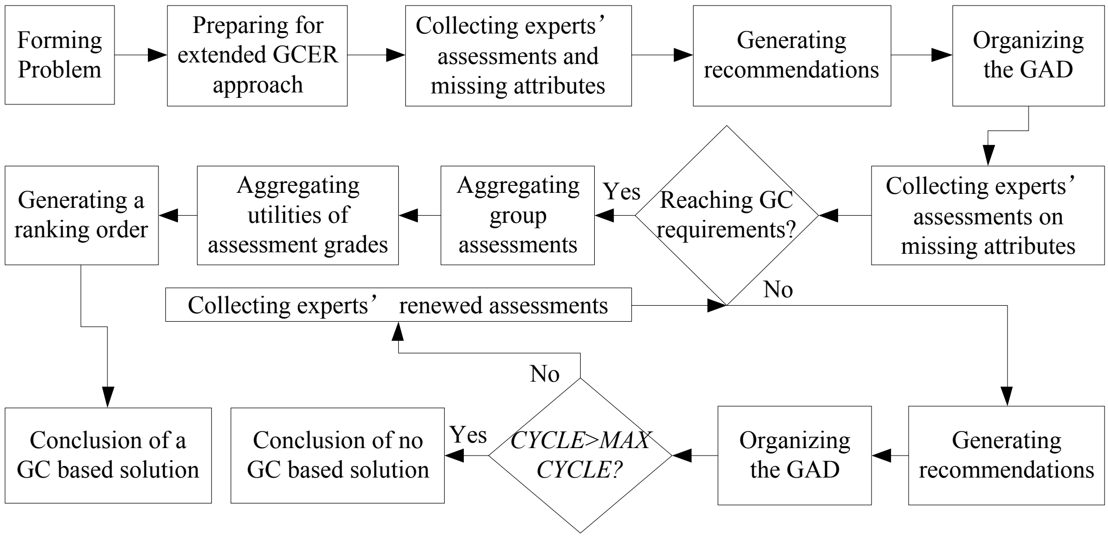

Figure 1. The procedure of GCER approach

Step 1: Form a MAGDA problem.

A manager selects T experts, identifies L basic attributes and their types (benefit or cost) and N assessment grades, and lists M alternatives to form a MAGDA problem.

Step 2: Prepare for the extended GCER approach in order to solve the MAGDA problem.

The manager specifies *MAXCYCLE*, the maximum times of GAD and assessments renewing to avoid the delayed convergence of collective solution and endless rounds of GAD and assessments renewing (Bryson, 1996; Herrera-Viedma et al., 2002; Choudhury et al., 2006; Mata et al., 2009); sets *CYCLE*=0, a cycle counter; decides the relative weights of L attributes; specifies a specific level, its corresponding threshold vector or threshold, and the thresholds ρ_1 and ρ_2 for its corresponding identification rule; and decides $sw^j(e_i)$ (j=1,…,T, i=1,…,L) and γ which are demonstrated in Sections A.1 and A.3 in Appendix A.

Step 3: Collect and unify experts' assessments, and collect missing attributes.

Experts independently give their assessments on attributes for alternatives and their utilities of assessment grades. The manager collects their assessments and unifies them using the unified utilities of assessment grades as mentioned in Section A.1, and collects missing attributes.

Step 4: Generate recommendations on missing attributes.

Generate recommendations to specific experts by reaching the maximum of GC on missing attributes for specific or all alternatives.

Step 5: Organize the GAD and collect experts' assessments on missing attributes.

The manager organizes the GAD and collects experts' assessments on missing attributes for specific or all alternatives based on recommendations according to experts' preferences.

Step 6: Decide whether the GC at the specific level is reached.

The manager decides whether the GC at the specific level is reached according to Equations (7)-(9). If so, go to Step 8. Otherwise, go to Step 7.

Step 7: Generate recommendations, organize the GAD and collect experts' renewed assessments.

If *CYCLE*>*MAXCYCLE* then go to Step 11; otherwise the recommendations to specific experts on how to renew assessments damaging to the GC at the specific level are generated according to the set of identification rules at the same level and the set of suggestion rules, *CYCLE*=*CYCLE*+1 is set by the manager, and the GAD is organized by the manager to help specific experts to renew their assessments. In the GAD, experts are free to communicate with each other and never provided by suggestions from the manager on their assessments to reach the GC. After that, go to Step 6.

Step 8: Form the aggregated group assessment on each alternative.

Calculate the aggregated group assessment on each alternative as mentioned in Section A.3.

Step 9: Obtain the aggregated utilities of assessment grades.

Obtain the aggregated utilities of assessment grades for alternatives according to Equation (A.14) in Appendix A.

Step 10: Generate a ranking order of *M* alternatives.

Generate a ranking order of *M* alternatives using the MRA (Wang et al., 2005, 2006b) when the maximum and minimum utilities of each alternative are derived from the aggregated assessments and utilities of assessment grades for alternatives.

Step 11: Finish the procedure.

The manager checks whether *CYCLE*>*MAXCYCLE* holds. If so, a conclusion of no GC based solution for the MAGDA problem can be drawn. Otherwise, the optimum alternative or the ranking order of *M* alternatives can be selected as a final solution to the MAGDA problem reaching the predefined GC.

4. Illustrative example

In this section, an engineering project management (EPM) software selection problem will be solved by the extended GCER approach as a real case to demonstrate its application to modeling a MAGDA problem, its detailed implementation process, and its validity.

A selfdeveloped solving system is used to effectively and efficiently solve the EPM software selection problem.

4.1. The description of the EPM software selection problem

Consider the EPM software selection problem with three software providers (SPs) which compete to provide their software and services for a famous Chinese automobile manufacturing enterprise. This can be considered a MAGDA problem, including a manager and five experts from the information department, the project management department, the financial department, the planning department, and the cooperator, and thirteen qualitative attributes shown in Table B.1 in Appendix B used to compare three SPs. The attributes are the attitude (AT), the technical support level (TSL), the implementing experience (IE), the core functions (CF), the peripheral functions (PF), the extended functions (EF), the validity of solution (VS), the after sales service and training (AST), the core requirements (CR), the interface requirements (IR), the security requirements (SR), the value for money of software expense (VMSE), and the value for money of consultation expense (VMCE).

The relative weights of the thirteen basic attributes are specified by the manager as w=(0.03, 0.05, 0.07, 0.09, 0.07, 0.04, 0.09, 0.06, 0.04, 0.04, 0.02, 0.2, 0.2).

Suppose three SPs are assessed by using the following set of assessment grades: *Poor (P)*, *Average (A)*, *Good (G)*, *VeryGood (V)*, and *Excellent (E)*, say

$\Omega=\{H_n, n=1,\ldots,5\}=\{Poor, Average, Good, VeryGood, Excellent\}=\{P, A, G, V, E\}$.

Thirteen attributes are all assessed by the above set of assessment grades.

The manager wants to obtain an optimum SP which is a GC based optimum choice for five experts in order to ponder over their opinions and find underlying problems as far as possible, which may significantly influence the validity of the optimum SP and the implementation effectiveness of its software and services.

The manager specifies that *MAXCYCLE* is equal to 4, identification thresholds ρ_1 and ρ_2 are equal to 0.3 and 0.45, respectively, and the GC is checked at the alternative level by a threshold δ_M =0.55; sets *CYCLE*=0; and decides γ=0.5 for a heterogeneous group of experts.

4.2. The collection of experts' assessments on missing attributes

Missing attributes for the first time of group assessment are shown in Table B.2 in Appendix B. By solving the optimization problems constructed on missing attributes with lingo software package, the recommendations are generated and shown in Table B.2. After the GAD organized by the manager, specific experts give their assessments on missing attributes for specific alternatives, which are shown in Table B.2, based on the recommendations according to their preferences.

4.3. The checking of the GC at the alternative level

To find a GC based solution to the EPM software selection problem, the GC at the alternative level must be reached first.

Five experts independently express their utilities of H_n (n=1,…,5) as shown in Table B.3 in Appendix B. How to elicit their utilities is not discussed here. Interested readers may refer to (Farguhar, 1984; Keeney and Raiffa, 1976; Winston, 1994; Zeleny, 1982) for details. Their relative importance on thirteen attributes is specified by the manager as also shown in Table B.4 in Appendix B.

Based on $u^j(H_n)$ (j=1,…,5, n=1,…,5) and $sw(e_i)$ (i=1,…,13), the unified utilities on thirteen attributes can be calculated according to Equation (A.2) in Appendix A, which are shown in Table B.5 in Appendix B.

After unifying experts' assessments using the above unified utilities, as stated in Section A.1, the GC on each attribute for each SP can be calculated, which are shown in Table B.6 in Appendix B. Further, the GC for three SPs can be obtained as

$gc(SP_1)$=0.5337, $gc(SP_2)$=0.5361 and $gc(SP_3)$=0.5494.

The GC at the alternative level for three SPs is clearly not reached. Therefore the recommendations generated by the set of identification rules at the alternative level and the set of suggestion rules in the feedback mechanism have to be given to specific experts. And the specific experts renew their assessments based on the recommendations after the GAD.

4.4. The recommendations in the feedback mechanism and assessments renewing

Based on experts' assessments in Table B.1 and renewed ones in Table B.2, the PMs at three levels are calculated according to Equations (10)-(12) and shown in Table B.7 in Appendix B. By use of GC and PMs at the attribute and alternative levels, the identification sets and recommendations can be generated, which are shown in Table B.8 in Appendix A. Specific experts independently renew their assessments based on recommendations after the first round of GAD with the feedback, i.e. *CYCLE*=1, which are also shown in Table B.8. After that, the GC on each attribute for each SP is recalculated and shown in Table B.9 in Appendix B. Further, the GC for three SPs can be obtained as

$gc(SP_1)$=0.6158, $gc(SP_2)$=0.673 and $gc(SP_3)$=0.5628.

The GC for three SPs is clearly reached. So a GC based solution for the EPM software selection problem can be found by reason of $CYCLE<MAXCYCLE$.

4.5. The achievement of the GC based solution

After the first round of recommendations, GAD and assessment renewing, the renewed assessments for the EPM software selection problem are firstly unified as those on $\bar{u}_i(H_n)$ (i=1,…,13, n=1,…,5). As demonstrated in Section A.3, $B(y(SP_l))$ (l=1,…,3) can be obtained, which is shown in Table 1.

Table 1

The aggregated belief degrees for three SPs

SPs	*P*	*A*	*G*	*V*	*E*	*Ω*
SP_1	0.0086	0.0374	0.1236	0.2839	0.5465	0
SP_2	0.0813	0.1665	0.3131	0.2448	0.1943	0

SP_3	0.004	0.043	0.1235	0.2939	0.5356	0

According to Equation (A.14), $u(H_n)$ (n=1,…,5) for three SPs can be calculated as 0, 0.2395, 0.531, 0.7832, and 1. Given $B(y(SP_l))$ and $u(H_n)$, the expected utilities and the ranking order of three SPs can be achieved as shown in Table 2 using the MRA. As a consequence, the manager obtains the optimum SP, SP_1, which is a GC based optimum choice for five experts.

Table 2

The expected utilities of three SPs and their ranking order

Expected utilities	SP_1	SP_2	SP_3
Minimum expected utility	0. 8434	0. 5922	0. 8416
Maximum expected utility	0. 8434	0. 5922	0. 8416
Average expected utility	0. 8434	0. 5922	0. 8416
Rank	1	3	2

5. Discussions

In (Fu and Yang, 2010), the GCER approach was compared with other GC based GDA approaches (e.g., Bordogna et al., 1997; Ben-Arieh and Chen, 2006; Dong et al., 2008, 2009; Choudhury et al., 2006; Szmidt and Kacprzyk, 2003; Herrera et al., 1996, 1997; Herrera-Viedma et al., 2005, 2007; Mata et al., 2009) and showed its advantages including the consideration of experts' utilities and subjective weights, the flexibility in consensus measures and the special design for MAGDA problems. These advantages are completely retained in the extended GCER approach.

Except for the advantages aforementioned, the extended GCER approach has two main contributions which are generating recommendations on missing attributes for specific or all alternatives and generating recommendations for specific experts in a feedback mechanism to renew assessments so as to accelerate the GC reaching process.

In many existing GDA approaches, the situation of incomplete (missing) assessments of experts has been already dealt with (e.g., Herrera-Viedma et al., 2007a, 2007b; Cabrerizo et al., 2010). Based on the property of additive transitivity, which can be seen as the parallel concept of the consistency property for Satty's multiplicative preference relation (Herrera-Viedma et al., 2004), Herrera-Viedma et al.' defined an additive consistency measure and used it to develop an iterative procedure to estimate the missing values of incomplete fuzzy preference relation (Herrera-Viedma et al., 2007a, 2007b). It can guarantee that the estimated assessment is only associated with the expert's originally given opinions and consistent with them. Furthermore, Cabrerizo et al. extended Herrera-Viedma et al.'s work in an unbalanced fuzzy linguistic context (Cabrerizo et al., 2010). Different from the above approaches, the assessments on missing attributes for specific alternatives in this paper are given by the expert after the GAD based on the recommendations generated by reaching the maximum of GC on missing attributes for specific alternatives. It will speed up the convergence of GC reaching process. At the same time, the preferences of the expert and other experts' directly given assessments on missing attributes for specific alternatives are considered simultaneously. Furthermore, the expert can study from others on missing attributes for specific alternatives by the GAD. Therefore it might be a good idea to let the expert decide the missing assessment based on the calculated value.

As to the feedback mechanism, Herrera-Viedma et al. considered the GC and the consistency of each expert simultaneously to generate recommendations to experts, which can make experts' opinions closer and avoid self-contradiction. In their feedback mechanism, a preference identification process and an advice process are designed to identify fuzzy preference values damaging to consistency/consensus state and generate recommended fuzzy preference values to specific experts, respectively (Herrera-Viedma et al., 2007a). Mata et al. further developed an adaptive consensus support model to identify the preference sets to be changed at the levels of pairs of alternative, alternative and preference relation, respectively, when the consensus at the preference relation level is decided to be very low, low and medium, respectively, and advise specific experts to increase, decrease or keep their fuzzy preference values (Mata et al., 2009). These existing feedback mechanisms undoubtedly help experts to reach the predefined GC. However, Herrera-Viedma et al.'s work cannot flexibly and efficiently identify fuzzy preference values to be changed when the GC has been gradually increased. Meantime, in Mata et al.'s work it might be a good idea to consider discriminatingly identifying the preference set when the GC at any of three levels rather than only at the preference relation level is required to be reached and increased from very low to medium state, like the set of identification rules at the attribute, alternative and global levels in this paper. In addition, the existing feedback mechanisms could add the GAD after the advices are given to experts to help them rationally renew assessments.

6. Conclusions

Focusing on evaluating assessments on missing attributes and speeding up the consensus reaching process in the GDA, this paper extended the GCER approach (Fu and Yang, 2010) to model MAGDA problems with missing assessments and GC requirements.

In the extended GCER approach, to reach the maximal GC on missing attributes for specific or all alternatives, optimization problems were constructed and solved to generate the recommendations on missing attributes, based on which experts gave their assessments according to their preferences after the GAD. With the PMs and GC at three levels, a set of flexible and efficient identification rules at any of three levels and a set of suggestion rules are developed to generate recommendations to help experts renew their assessments when the GC at any of three levels is required to be reached. The effectiveness and applicability of the extended GCER approach was demonstrated by solving an EPM software selection problem.

Acknowledgements

This research is supported by the National Natural Science Foundation of China (no. 70631003, 90718037 and 70801024), and the Specialized Research Fund for the Doctoral Program of Higher Education of MOE of China (no. 200803590007).

References

Alonso, S., Chiclana, F., Herrea, Herrera-Viedma, F.E., Alcala-Fdez, J., Porcel, C.,

2008. A consistency-based procedure to estimate missing pairwise preference values. International Journal of Intelligent Systems 23(2), 155-175.

Ben-Arieh, D., Chen, Z.F., 2006. Linguistic-labels aggregation and consensus measure for autocratic decision making using group recommendations. IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Human 36 (3), 558-568.

Bordogna, G., Fedrizzi, M., Pasi, G., 1997. A linguistic modeling of consensus in group decision making based on OWA operators. IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans 27 (1), 126-132.

Bryson, N., 1996. Group decision-making and the analytic hierarchy process: Exploring the consensus-relevant information content. Computers and Operations Research, 1 (23), 27–35.

Cabrerizo, F.J., Alonso, S., Herrera-Viedma, E., 2009. A Consensus Model for Group Decision Making Problems with Unbalanced Fuzzy Linguistic Information. International Journal of Information Technology & Decision Making 8 (1), 109-131.

Cabrerizo, F.J., Perez, I.J., Herrera-Viedma, E., 2010. Managing the consensus in group decision making in an unbalanced fuzzy linguistic context with incomplete information. Knowledge-Based Systems 23, 169-181.

Chin, K.S., Wang, Y.M. Poon, G.K., Yang, J.B., 2009. Failure mode and effects analysis using a group-based evidential reasoning approach. Computers & Operations Research 36(6), 1768-1779.

Chin, K.S., Xu, D.L., Yang, J.B., Lam, J.P.K., 2008. Group-based ER-AHP system for product project screening. Expert Systems with Applications 35(4), 1909-1929.

Choudhury, A.K., Shankar, R., Tiwari, M.K., 2006. Consensus-based intelligent group decision-making model for the selection of advanced technology. Decision Support Systems 42, 1776-1799.

Dempster, A.P., 1967. Upper and lower probabilities induced by a multi-valued mapping. Annals of Mathematical Statistics 38, 325-339.

Denoeux, T., 1999. Reasoning with imprecise belief structures. International Journal of Approximate Reasoning 20, 79-111.

Denoeux, T., 2008. Conjunctive and disjunctive combination of belief functions induced by non distinct bodies of evidence. Artificial Intelligence 172 (2-3), 234-264.

Dong, Y.C., Xu, Y.F., Li, H.Y., 2008. On consistency measures of linguistic preference relations. European Journal of Operational Research 189, 430-444.

Dong, Y.C., Xu, Y.F., Li, H.Y., Feng, B., 2009. The OWA-based consensus operator under linguistic representation models using position indexes. European Journal of Operational Research.

Doyle, J.R., Green, R.H., Bottomley, P.A., 1997. Judging relative importance: direct rating and point allocation are not equivalent. Organizational Behavior and Human Decision Processes 70, 55-72.

Farguhar, P.H., 1984. Utility assessment methods. Management Science, 30 (11), 1280-1300.

Fu, C., Yang, S.L. The group consensus based evidential reasoning approach for multiple attributive group decision analysis. European Journal of Operational Research (2010), doi:10.1016/j.ejor.2010.03.011.

Guo, M., Yang, J.B., Chin, K.S., Wang, H.W., 2007. Evidential reasoning based preference programming for multiple attribute decision analysis under uncertainty. European Journal of Operational Research 182, 1294-1312.

Herrera, F., Herrera-Viedma, E., Verdegay, J.L., 1996. A model of consensus in group decision making under linguistic assessments. Fuzzy Sets and Systems 78, 73-87.

Herrera, F., Herrera-Viedma, E., Verdegay, J.L., 1997. A rational consensus model in group decision making using linguistic assessments. Fuzzy Sets and Systems 88, 31-49.

Herrera-Viedma, E., Alonso, S., Chiclana, F., Herrera, F., 2007a. A consensus model for group decision making with incomplete fuzzy preference relations. IEEE Transactions on Fuzzy Systems 15 (5), 863-877.

Herrera-Viedma, E., Chiclana, F., Herrera, F., Alonso, S., 2007b. Group decision-making model with incomplete fuzzy preference relations based on additive consistency. IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics 37 (1), 176-189.

Herrera-Viedma, E., Herrera, F., Chiclana, F., 2002. A consensus model for multiperson decision making with different preference structures. IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Human 32 (3), 394–402.

Herrera-Viedma, E., Herrera, F., Chiclana, F., Luque, M., 2004. Some issues on consistency of fuzzy preference relations. European Journal of Operational Research 154, 98-109.

Herrera-Viedma, E., Martinez, L., Mata, F., Chiciana, F., 2005. A consensus support system model for group decision-making problems with multigranular linguistic preference relations. IEEE Transactions on Fuzzy Systems 13, 644-658.

Hwang, C.L., Lin, M.J., 1987. Group Decision Making Under Multiple Criteria. Lecture Notes in Economics and Mathematical Systems 281, Springer-Verlag, Berlin/Heidelberg.

Hwang, C.L., Yoon, K., 1981. Multiple attribute decision-making: methods and applications. Springer, Berlin.

Keeney, R.L., Raiffa, H., 1976. Decision with multiple objective: preference and value tradeoffs. Wiley, New York.

Kim, S.H., Ahn, B.S., 1999. Interactive group decision making procedure under incomplete information. European Journal of Operational Research, 116(3): 498-507.

Kim, S.H., Han, C.H., 1999. An interactive procedure for multi-attribute group decision making with incomplete information. Computers and Operations Research, 26: 755-772.

Kim, S.H., Choi, S.H., Kim, J.K., 1999. An interactive procedure for multiple attribute group decision making with incomplete information: range-based approach. European Journal of Operational Research, 118(1): 139-152.

Li, D.F., Yang, J.B., 2004. Fuzzy linear programming technique for multiattribute group decision making in fuzzy environments. Information Sciences 158, 263-275.

Linstone, H.A., Turoff , M., (Eds.), 1975. The Delphi Method; Techniques and Applications. Addison Wesley.

Liu, W., 2006. Analyzing the degree of conflict among belief functions. Artificial Intelligence 170, 909-924.

Mata, F., Martínez, L., Herrera-Viedma, E., 2009. An Adaptive Consensus Support Model for Group Decision Making Problems in a Multi-Granular Fuzzy Linguistic Context. IEEE Transactions on Fuzzy Systems 17 (2), 279-290.

Olcer, A.I., Odabasi, A.Y., 2005. A new fuzzy multiple attributive group decision making methodology and its application to propulsion/manoeuvring system selection problem. European Journal of Operational Research 166, 93-114.

Roberts, R., Goodwin, P., 2002. Weight approximations in multi-attribute decision analysis. Journal of Multi-Criteria Decision Analysis 11, 291-303.

Shafer, G., 1976. A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jeresy.

Smets, P., 1995. The canonical decomposition of a weighted belief, in: Int. Joint Conf. on Artificial Intelligence, Morgan Kaufman, San Mateo, CA, pp. 1896-1901.

Smets, P., 2004. Decision making in the TBM: the necessity of the pignistic transformation. International Journal of Approximate Reasoning 38, 133-147.

Smets, P., Kennes, K., 1994. The transferable belief model. Artificial Intelligence 66 (2), 191-234.

Srinivasan, V., Shocker, A.D., 1973. Linear programming techniques for multidimensional analysis of preferences. Psychometrika 38, 337-369.

Szmidt, E., Kacprzyk, J., 2003. A consensus-reaching process under intuitionistic fuzzy preference relations. International Journal of Intelligent Systems 18, 837-852.

Takeda, E., Cogger, K.O., Yu, P.L., 1987. Estimating criterion weights using eigenvectors: a comparative study. European Journal of Operational Research 29, 360-369.

Tavana, M., Kennedy, D.T., Joglekar, P., 1996. A group decision support framework for consensus ranking of technical manager. Omega 24 (5), 523-538.

Wang, Y.M., Greatbanks, R., Yang, J.B., 2005. Interval efficiency assessment using data envelopment analysis. Fuzzy Sets and Systems 153 (3), 347-370.

Wang, Y.M., Yang, J.B., Xu, D.L., 2006a. Environmental impact assessment using the evidential reasoning approach. European Journal of Operational Research 174, 1885-1913.

Wang, Y.M., Yang, J.B., Xu, D.L., Chin, K.S., 2006b. The evidential reasoning approach for multiple attribute decision analysis using interval belief degrees. European Journal of Operational Research 175, 35-66.

Winston, W., 1994. Operations research applications and algorithms. Duxburg Press, California.

Wu, Z.B., Chen, Y.H., 2007. The maximizing deviation method for group multiple attribute decision making under linguistic environment. Fuzzy Sets and Systems 158, 1608-1617.

Yang, J.B., 2001. Rule and utility based evidential reasoning approach for multiattribute decision analysis under uncertainties. European Journal of Operational Research 131, 31-61.

Yang, J.B., Sen, P., 1994. A general multi-level evaluation process for hybrid MADA with uncertainty. IEEE Transactions on Systems, Man, and Cybernetics 24 (10), 1458-1473.

Yang, J.B., Sen, P., 1997. Multiple attribute design evaluation of large engineering products using the evidential reasoning approach. Journal of Engineering Design 8 (3), 211-230.

Yang, J.B., Singh, M.G., 1994. An evidential reasoning approach for multiple attribute decision making with uncertainty. IEEE Transactions on Systems, Man, and Cybernetics 24 (1), 1-18.

Yang, J.B., Wang, Y.M., Xu, D.L., Chin, K.S., 2006. The evidential reasoning approach for MADA under both probabilistic and fuzzy uncertainties. European Journal of Operational Research 171, 309-343.

Yang, J.B., Xu, D.L., 2002a. On the evidential reasoning algorithm for multiattribute decision analysis under uncertainty. IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans 32 (2), 289-304.

Yang, J.B., Xu, D.L., 2002b. Nonlinear information aggregation via evidential reasoning in multiattribute decision analysis under uncertainty. IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans 32 (3), 376-393.

Zeleny, M., 1982. Multiple criteria decision making. McGraw-Hill, New York.