A STUDY OF SUBPIXEL REGISTRATION ALGORITHMS

by
Qi Tian and Michael N. Huhns

Center for Machine Intelligence
University of South Carolina
Columbia, SC 29208

ABSTRACT

This paper presents an analysis of four algorithms which are able to register images with subpixel accuracy: these are correlation interpolation, intensity interpolation, differential method, and phase correlation. The main factors affecting registration accuracy are the interpolation function, sampling frequency, number of bits per pixel, and frequency content of the image; these are analyzed and evaluated by computer simulations and measuments of object translation. Using bilinear interpolation and representing pixels by 8-bit samples, a 0.01 to 0.05 pixel registration accuracy can be achieved. The four registration algorithms can be ordered according to accuracy as follows; intensity interpolation, difference method, correlation one-dimensional interpolation, correlation two-dimensional interpolation and phase correlation. Moreover, these are also compared in terms of computational cost and ease of use.

1. Algorithms for subpixel Registration

Subpixel registration has emerged from different application areas. In some application areas it is generally acceptable to achieve a registration result with a distance accuracy of ± 1 pixel; but in some problems, such as high accuracy registration and time-varying image analysis, registration results with a higher accuracy than one pixel distance are essential. In TV image sequence analysis, several successive frames of images are analyzed and motions of objects are estimated with a subpixel accuracy by using a difference method. This estimation is further used to restore and enhance the quality of TV images [1]. In the instrumentation area, the subpixel method is used to improve registration accuracy by using the correlation interpolation technique; it is reported that under some assumptions 0.05 pixel accuracy has been achieved [2]. In nondestructive evaluation, several frames of images of an object during translation or deformation are taken and analyzed. Any interesting point of the object can be detected to obtain its translation with a 0.1 to 0.05 pixel accuracy [3]. These methods have also been used to calculate fluid velocity distributions [4]. Having numerous valuable applications, this has led to increased research in subpixel registration.

1.1. Review of Subpixel Registration Algorithms

This Section describes four subpixel registration algorithms: correlation interpolation, intensity interpolation, differential method, and phase correlation method.

When the sampling frequency of images is high enough, the corresponding discrete correlation function is quite smooth. Therefore it can be interpolated by using a second-order equation, and the resulting surface can be searched to locate a peak position accurately [2]. This subpixel estimation reduces the line off-set estimation error from about 0.5 pixel to less than 0.1 pixel (worst case).

Subpixel registration can be achieved by using intensity interpolation to create a new version of the reference image with a much denser intensity grid. A search using the target image is then conducted over this new reference image for every possible position [3]. For example, given target image M x M and reference image N x N, if an accuracy of 0.1 pixel is desired for registration then the reference image should be interpolated to create a new version of the reference image with a (10 x N) x (10 x N) dimension. Then, a search should cover every position of (10 x N - M + 1)**2.

A differential method is used for estimating two-dimensional translations to process a sequence of TV images in [1]. The key idea is to relate the difference of two successive frame images to the intensity spatial gradient of the first image. Given two images, $f_1(x,y)$ and $f_2(x,y)$, assume that translations of an object centered at location (x,y) of image 1 with respect to image 2 are Dx and Dy, respectively, then the frame difference can be expressed as

$$I(x,y) = f_1(x,y) - f_2(x,y)$$

$$= \frac{\partial f_1(x,y)}{\partial x} Dx + \frac{\partial f_1(x,y)}{\partial y} Dy$$

where $\partial f_1(x,y)/\partial x$ and $\partial f_1(x,y)/\partial y$ are the partial gradients of $f_1(x,y)$.

The phase correlation technique has been used to implement a video-rate image correlation processor [5]. It achieves subpixel accuracy with relative insensitivity to scene content, illumination differences, and narrow-band noise. It is based on the fact that most of the information about the relative displacements of objects between two images is contained in the phase of their cross-power spectrum. It was reported that the algorithm provides results with a 0.08 pixel accuracy [5]. The advantage of the method is that it can be used when images are seriously distorted, in either geometry or intensity.

1.2. Advantages of Subpixel Registration

The advantages of subpixel registration can be explored in several aspects. In some applications, the high accuracy achieved by using subpixel registration also can be obtained using pixel-level registration, provided that high-resolution images are used for the registration. However, this requires increases in memory size, computational time, and hardware cost. For example, in order to achieve a 0.1 pixel accuracy for an image of size 512 x 512, pixel-level registration requires that a 5120 x 5120 image be taken and stored. Obviously, it would take 100 times more memory and the search space would have to be 100 times bigger. Therefore, in terms of computational load and memory size, subpixel registration is much more efficient than pixel-level registration. Furthermore, in some applications it is impossible to obtain images that have the necessary resolution, then the subpixel registration method is the only solution.

2. Accuracy Analysis and Comparison of the Subpixel Registration Algorithms

Computer simulations by using computer-generated images, and experiments for measuring objects' displacements using speckle images have been conducted to evaluate the accuracy of the intensity interpolation algorithm, and to compare it with other algorithms. In the simulation, to avoid noise and distortion problems, one-dimensional signals and two-dimensional images are computer-generated. In this way, the translation between two given signals is exactly predetermined. Thus, test results can be compared with the given translations. Errors indicate the algorithms accuracies. The experimental set-up for measuring displacements of objects are described in [3].

2.1 Accuracy Estimation of Intensity Interpolation Algorithm

Theoretically, if a sampling frequency is high enough, a sinc interpolation function is used to restore the original signal, and the given digitized signal is represented using enough bits to accurately represent the original signal, then the intensity interpolation algorithm can provide accurate registration results. But, in practice, errors are introduced because the three above assumptions, more or less, are not insured in subpixel registration.

A sampling frequency below the Nyquist frequency causes aliasing errors. In our experiments, a one-dimensional sawtooth waveform has been used to demonstrate this effect. Two-point average and three-point average low pass filters were used to remove high frequencies, which occur near the sharp points on the waveforms. Table 1 indicates that 1) the accuracy can be improved by one order, and 2) the performance of the three-point average filter is much better than the two-point average filter. In this simulation, signals of real values were used to test the effects of filtering independent of effects of finite-length words.

Secondly, the finite number of bits per pixel adds to the registration error. To test the magnitude of this error, two straight lines with a slope equal to 5, are shifted from each other by 0.611111 pixel distance and then sampled. If both signals are represented by real-values sampled then the test error is 0.000005 pixel; if they are represented by an 8-bit integer then the error is about 0.01 pixels. For a set of sine waveforms, errors for using real and integer signals are 0.0005 and 0.003, respectively. This means that in order to achieve registration accuracies better than 0.01 pixel more than 8 bits is required to represent each sample.

In the experiments measuring displacements of objects, before moving the table we digitized an image, then moved the table by a certain distance in the x direction and digitized the second image. In the same way, other images were continually digitized. The measured results for displacements by using the iterative intensity interpolation are shown in Table 3. The accurate displacement in the x direction for each step is 0.001 inch and for the y direction is 0.000 inch. The experiments show that by using the intensity interpolation algorithm an accuracy of 0.01 to 0.05 pixels can be achieved using speckle images to measure displacements of objects.

The above results were obtained by using bilinear interpolation and no attempt has been made to try other interpolation algorithms. Nearest-neighbor interpolation can not be used for the subpixel registration because it causes a half-pixel shift. Higher-order interpolation may improve the accuracy. However, based on a simulation, bilinear interpolation which achieves an accuracy of about 0.005 pixel is already very time-consuming. Also, in practice, the accuracy is mainly limited by system noise, not by the interpolation method.

In summary, computer simulations and experiments measuring the displacements of real objects indicate that by using bilinear interpolation, 8 bits per pixel, and a sampling rate greater than the Nyquist frequency, an 0.01 to 0.05 pixel accuracy can be achieved.

2.2. Comparison of Intensity Interpolation with Other Algorithms

Let's compare all algorithms for subpixel registration in terms of accuracy, speed and usage.

The correlation interpolation method seems to be the easiest way to implement subpixel registration. Unfortunately, it is not ideal. In a one-dimensional case if correlation curves have a known approximate mathematical expression, then using a second-order interpolation peak position can be calculated accurately. In a two-dimensional case there are several methods which can be used to find an approximate location of a peak. A 5-point, 6-point or 9-point neighborhood near the peak can be used to fit a surface which approximates the correlation surface; then, from the mathematical expressions of the surfaces, positions of peaks can be located. Separate one-dimensional interpolations in the X and Y directions, instead of two-dimensional interpolation, can be also used to determine two coordinates of the peak position. Experiments show that in a two-dimensional case the separable interpolations are better than the two-dimensional one. Overall, in terms of accuracy, correlation interpolation is less accurate than the intensity interpolation method while having a much lower computational cost. A set of experimental data is listed in Table 2.

The difference method is mainly used to process image frames where objects only have small translations, such as successive television frames of slowly moving objects. It is because the algorithm preassumes that during translation the intensity gradients of the images do not change. The difference method can be used when there are several objects with different translation direction and speeds. Computational costs depend on the dimensions of the area which is used for calculation. For example, the area can be 4 x 4 pixels to 64 x 64 pixels, depending on individual problems. Compared with other methods, its computational cost is the lowest. Its usage is limited to the small displacement case. Generally, its accuracy is lower than the intensity interpolation method, but higher than the correlation interpolation, as indicated in Table 2.

The phase-correlation method is only used when an entire image frame is uniformly shifted. If an image has several objects moving in different directions, then this method fails. Because it uses phase correlation of images to determine translations, the algorithm is not sensitive to either geometric distortions or noise. Experiments show that even when two images have less than 70% common area, the algorithm still provides satisfactory results. This is the reason why this method has been used for air-borne guidance systems. The accuracy is the lowest compared with other subpixel registration algorithms.

A sinc-like two-dimensional computer-generated images are used to test accuracy of different algorithms. Table 2 shows that the intensity interpolation algorithm provides the highest accuracy, followed by the difference method, correlation one-dimensional interpolation, correlation two-dimensional interpolation, and phase correlation.

Therefore, in terms of accuracy, intensity interpolation is the best, but it is also the most time-consuming among the above algorithms. Although using an iterative method [3] dramatically reduces the number of computations needed, the cost is still not acceptable. In [6], a hill-climbing algorithm is incorporated with the intensity interpolation method, and a factor of 10 improvement in speed is achieved without effecting on the accuracy.

3. Conclusion

Subpixel registration can be used in a variety of application areas because it provides a way to accurately measure displacements of individual points of a plane, without any contact and disturbance. To our knowledge, a method with these characteristics has not been available before.

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Table 1 A set of test data to evaluate prefiltering

	Displacement=0.6111	Test Error
no filtering	0.6186	0.0075
two-point filtering	0.6149	0.0038
three-point filtering	0.6106	0.0005

Table 2 Test results of different algorithms applied to sinc-like image pair.

	intensity interpolation	differ.	correla. 1-D inter.	correla. 2-D inter.	phase
Xerr.	.0029	.013	.043	.047	.092
Yerr.	.006	.010	.035	.069	.127

Table 3 Measurement results using speckle images (values in pixels)

Actual	0.310	3.100	4.650	6.200	7.750	9.300
Measured	0.326	3.025	4.607	6.145	7.761	9.338
Error	0.016	0.048	0.043	0.055	-0.011	-0.038