

# A FAST ITERATIVE HILL-CLIMBING ALGORITHM FOR SUBPIXEL REGISTRATION

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## ABSTRACT

A fast iterative hill-climbing algorithm has been developed for subpixel registration. It is illustrated by measuring the displacement of a two-dimensional object using two successive speckle images of the object. The algorithm yields an accuracy up to 0.05 pixel and a speed 10 times faster than a commonly-used iterative algorithm.

## 1. INTRODUCTION

Based on statistical properties of the cross-correlation functions of certain images, the shapes of global peaks on these functions can then be determined. Subpixel registration can be performed with significant computational savings in two steps by 1) locating an approximate position of the global peak by a proposed coarse search and 2) finding the global peak with subpixel accuracy by using hill-climbing.

## 2. A COARSE SEARCH BASED ON SHAPE OF CORRELATION OF IMAGES

To clarify this procedure a typical speckle image and its two-dimensional normalized cross-correlation function are presented in Figure 1-2. The speckle image looks like randomly scattered spots with similar diameters. The cross-correlation function has a global peak which corresponds to the correct registration position, and many local maxima, but there are no adjacent peaks located nearer than 6-8 pixels. As indicated in Figure 3, the peak width depends on the average spot diameter of the speckle image, and the two speckle images shown have 6 and 8-pixel diameters, respectively.

A capture region of the global peak is defined to be a small surrounding region of the peak where the cross-correlation function is unimodal[1]. The function's shape depends on the spatial content of the image, but an average width of the capture region, denoted by  $L$ , can be defined in terms of the area of the global peak.

Whenever any point inside the capture region has been determined, then the global peak can be located by using hill-climbing. The proposed coarse search is in the sense that the preliminary search is only conducted on a predetermined rectangular grid, the size of which is determined according to the width of the global peak. After completing the coarse search, the location of the global peak is known with an accuracy of  $L/2$ .

Thus, the search grid,  $K$ , should be equal to or smaller than half of the width of the capture region in order to correctly locate the global peak in the coarse search. Because the width of the capture region is related to the shape of the cross-correlation and this depends on the spatial content of the image, then a proper search-grid size can be determined based on statistical properties of the shapes of the cross-correlations for certain images. When the subimage size is chosen to be smaller, then the cross-correlation surface fluctuates more. In this case, the grid should be denser. Also, the subimage should include features which distinguish it from other parts of the image; for an image with a greater frequency content, like a speckle image, a subimage with a smaller size,  $18 \times 18$ , may be used in the coarse search. For images with fewer high frequencies, a bigger subimage should be used for the coarse search. For speckle images we can simply use  $1/2$  to  $1/3$  of the average size of the spots as the search grid spacing.

Figure 4 shows an  $M \times M$  reference image and an  $N \times N$  subimage taken from a translated image. The coarse search only searches a grid of size  $K \times K$  pixels.

For an exhaustive search the correlation has to be computed  $(M - N + 1)^2$  times[2]. The proposed coarse search only requires  $(M - N)^2/K^2$  computations. Thus there is a factor of  $K^2$  fewer computations, where for the speckle images of our experiment,  $K = 3$ . The bigger the search grid, the faster the computation becomes; but the cost is a high risk of missing the global peak in the

coarse search. Therefore, K has to be determined according to the width of the global peak.

### 3. AN ITERATIVE HILL-CLIMBING ALGORITHM

After the coarse search, an approximate location of the global peak is known in terms of the displacement between two images, denoted by  $D_x$  and  $D_y$ , for a capture region width  $L$ . A hill-climbing procedure next determines a precise location of the global peak in a square displacement domain centered at  $(D_x, D_y)$  and with sides of width  $L$ . The accuracy of the final location should be predetermined according to the algorithm and overall system accuracy limits.

In order to achieve subpixel registration accuracy, a new version of the reference image has to be created using interpolation, potentially resulting in many times as many pixels as the original image. The total number of pixels for this new version depends on the required final registration accuracy. In order to save computations for interpolation and subpixel registration, an iterative algorithm is used to perform this search. A detailed algorithm is described in [3,5]. Instead of directly creating a new version of the image with full resolution, a version is first calculated according to the displacement matrix, indicated in Figure 5a, with a resolution of  $L/10$ . After a search at this level, a more accurate location of the global peak is located at, for example,  $(D_{x1}, D_{y1})$ . Then a second version of the reference image is created by interpolation according to the displacement matrix (shown in Figure 5b) which is centered at  $(D_{x1}, D_{y1})$  and has a 10 times higher resolution than the first matrix. This is repeated until a predetermined resolution of the global peak location is reached. Here an absolute difference is used as a correlation measure, because near the registration position its resultant shape is much sharper than that of the normalized cross-correlation function. Thus it is advantageous for subpixel registration.

The hill-climbing strategy can be implemented in three ways. A corner-start search begins from one of the four corners. Each time values are calculated at three neighbor points, along the direction from this corner to the center. The point with minimum correlation value is chosen as the next starting position. This procedure is repeated until a point is reached where all its three neighbors have higher correlation values than it has.

The second method is a center-start search. Because the peak is more likely located near the center of the displacement

matrix, it would be preferable to begin searching from the center point instead of starting from the corners. To begin with, the center and its four neighbors are calculated, then the point with minimum correlation value is chosen as the next starting point. In Figure 6 a typical search pattern for the center-start search is presented.

The third method is a correlation non-increasing search. The climbing begins with one direction, either  $x$  or  $y$ , and proceeds along it until the correlation value begins to increase, then search proceeds in the other direction. The search pattern appears saw-like.

The computational loads of these three methods have been studied, and analyses indicate that the center-start search is best in terms of computational load and reliability.[3]

### 4. EXPERIMENTS AND APPLICATION AREAS

A hard-plastic board, firmly attached to an X-Y translation table, is illuminated by a He-Ne laser. It is translated by steps of 0.001 inches and 0.005 inches in the two experiments, respectively. A camera digitizes the images at 310 pixels per inch. Experiments show that errors between measured results and actual translations are smaller than 0.05 pixels[3]; the system's overall accuracy is constrained by interference in the synchronizing signals produced by the video digitizing system. The average error in the vertical direction is about 0.03 pixels while the error in the horizontal direction is about 0.01 pixels. The algorithm has been also applied to a pair of computer-created images with a known translation; results show that the algorithm, using a bilinear interpolation, can achieve an accuracy about 0.002 pixels. For measuring displacements of 20 points on an object, the iterative method takes 108.0 seconds while the center-start only 8.94 seconds, speeding up the computation more than 12 times. The corner-start and non-increasing methods take 20.3 and 14.5 seconds respectively.

This algorithm may be used in a variety of application areas where two-dimensional displacements need to be measured as accurately as possible without any contact and disturbance of the object. These include nondestructive evaluations, circuit-board alignment and VLSI lay-out.

### REFERENCES

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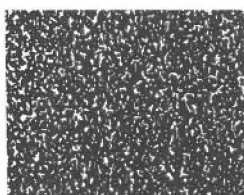


Figure 1. Speckle image with spot size = 8 pixels.

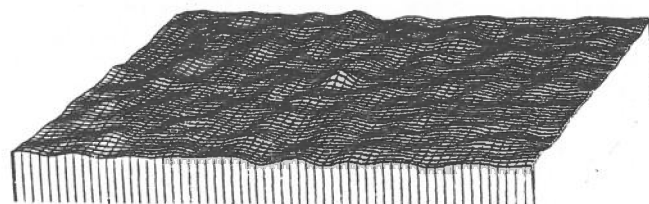


Figure 2. The 2-D cross-correlation of the speckle image for  $M = 18$ .

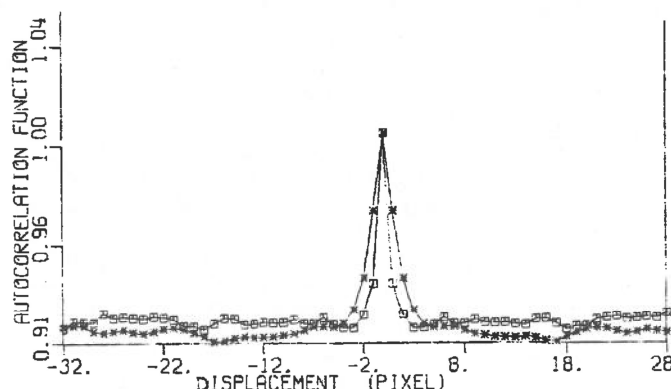


Figure 3. Autocorrelation of speckle images with different spot widths.

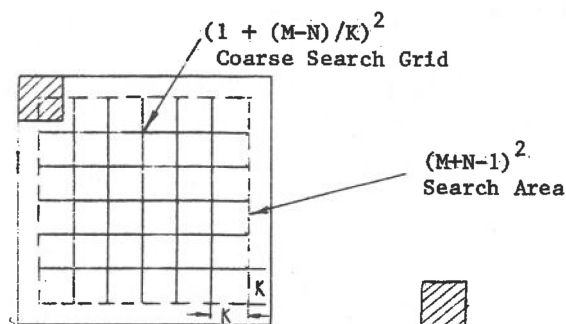
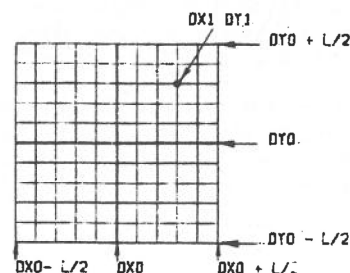
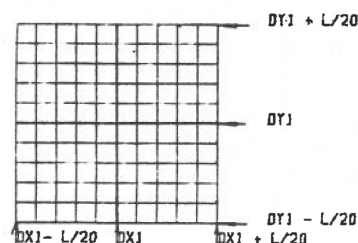


IMAGE IS:  $M \times M$ ; SUBIMAGE IS  $N \times N$

Figure 4. The image, subimage and coarse search grid.



Q. DISPLACEMENT VALUES FOR ITERATION 1



b). DISPLACEMENT VALUES FOR ITERATION 2

Figure 5. Search matrix of displacements for two iterations.\*

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ITERATION      1
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 4.5 3.8 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 7.0 5.5 4.3 3.5 3.7 0.0 0.0 0.0
0.0 0.0 0.0 0.0 8.8 8.0 6.5 5.4 4.9 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 8.8 7.8 7.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

THE MINIMUM CORRELATION IN U= 1.6000   V=-1.2000

ITERATION      2
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 3.359 3.355 0.000
0.000 0.000 0.000 0.000 0.000 0.000 0.000 3.382 3.359 3.346 3.342 3.347
0.000 0.000 0.000 0.000 0.000 0.000 3.417 3.384 3.344 3.352 3.349 0.000
0.000 0.000 0.000 0.000 0.000 3.480 3.440 3.410 3.389 3.378 0.000 0.000
0.000 0.000 0.000 0.000 3.520 3.482 3.453 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 3.579 3.542 3.514 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.000 3.619 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000

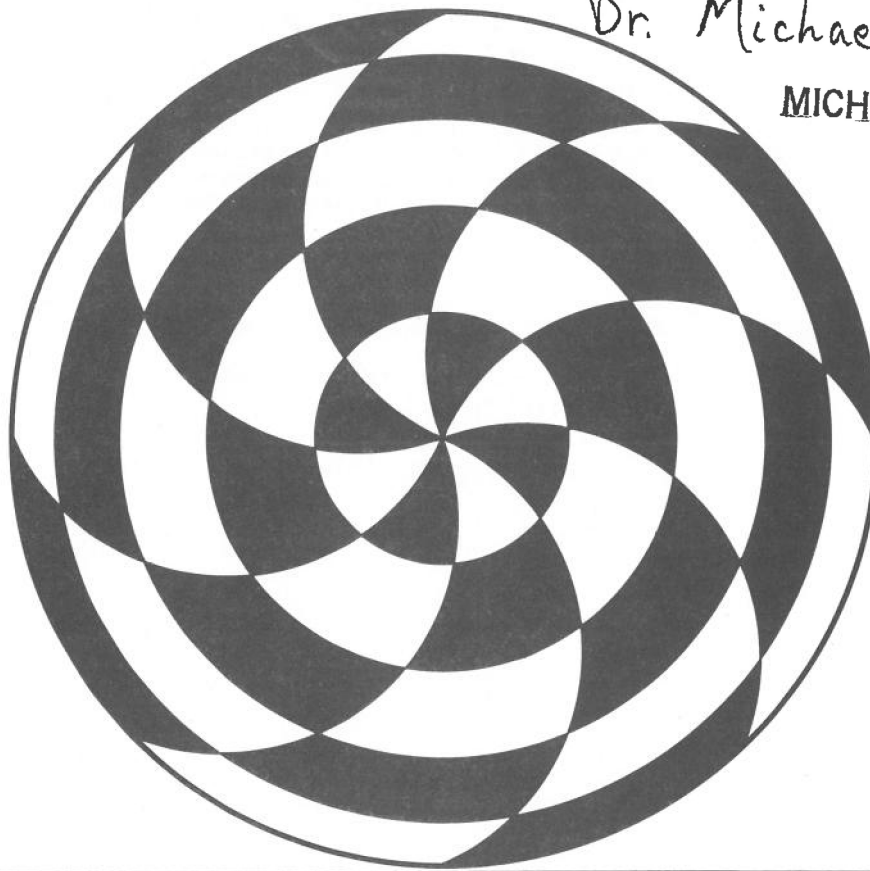
THE MINIMUM CORRELATION IN U= 1.6800   V=-1.2800

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Figure 6. A center-start two-iteration search.

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