Lecture 8

Karnaugh Map (K-map)

The method we learned in Ch. 2 has a problem…

As many of you may have seen in the homework, it doesn’t tell us if the result is a minimum

K-map

Karnaugh maps are a graphical approach for finding suitable product terms for use in SOP expressions

- Very useful for problems with 3 or 4 variables

Intro to K-map

K-maps consist of one square for each possible minterm in a function

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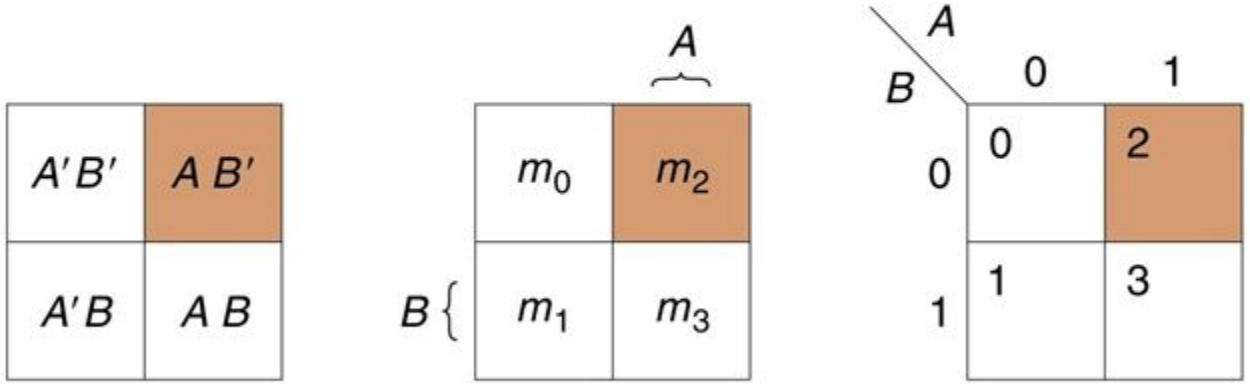
K-maps consist of one square for each possible minterm in a function

- 2 variable - 4 squares (2^2)
- 3 variable - 8 squares (2^3)
- 4 variable - 16 squares (2^4)
- n variable - 2^n squares

Example: 2 variable K-map

Example: 2 variable K-map

Map 3.1 Two-variable Karnaugh maps.



Example: 3 variable K-map

Example: 3 variable K-map

Notice that the last 2 columns are not in numerical order

If we organize the map this way, instead of in numerical order, the minterms in adjacent squares can be combined using the adjacency property

3 variable K-map

Map 3.3 Three-variable maps.

C \ AB	A′B′ 00	A′B 01	AB 11	AB′ 10
C′ 0	A′B′C′	A′BC′	ABC′	AB′C′
C 1	A′B′C	A′BC	ABC	AB′C

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

4 variable!

4 variable!

Map 3.8 The four-variable map.

C D \ A B	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

C D \ A B	00	01	11	10
00	$A'B'C'D'$	$A'B\,C'D'$	$A\,B\,C'D'$	$A\,B'C'D'$
01	$A'B'C'D$	$A'B\,C'D$	$A\,B\,C'D$	$A\,B'C'D$
11	$A'B'C\,D$	$A'B\,C\,D$	$A\,B\,C\,D$	$A\,B'C\,D$
10	$A'B'C\,D'$	$A'B\,C\,D'$	$A\,B\,C\,D'$	$A\,B'C\,D'$

Plotting a function

We will use minterms of a function and plot each square corresponding to each minterm

- Example: ab + ab’ = ?

More examples

How would we plot this?

- F = AB’ + AC + A’BC’

Implicant

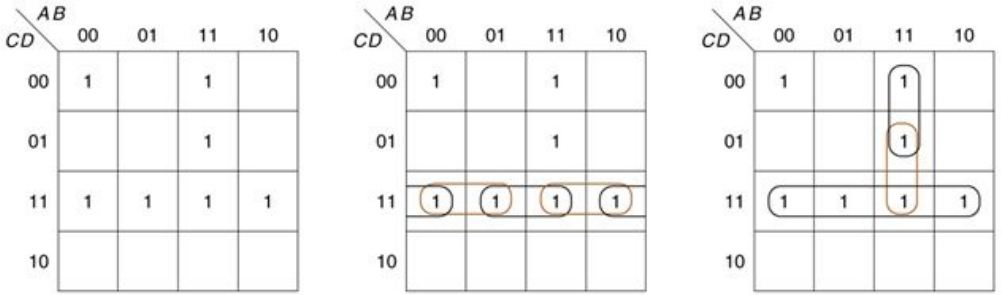
An implicant of a function is a product term that can be used in an SOP expression for that function

From the point of view of the map, an implicant is a rectangle of 1,2,4,8...2^n 1's. No 0's may be included

Implicant Example

Implicant Example

Map 3.12 A function to illustrate definitions.



The implicants of *F* are

Minterms	*Groups of 2*	*Groups of 4*
A´B´C´D´	*A´CD*	*CD*
A´B´CD	*BCD*	
A´BCD	*ACD*	
ABC´D´	*B´CD*	
ABC´D	*ABC´*	
ABCD	*ABD*	
AB´CD		

Prime Implicant

A prime implicant is an implicant that (from the point of view of the map) is not fully contained within any one other implicant

Essential Prime Implicant

An essential prime implicant is a prime implicant that includes at least one 1 that is not in any other prime implicant.

Why do we need these?

The map is supposed to help us find minimum SOP expressions

The only product terms we need to care about are prime implicants

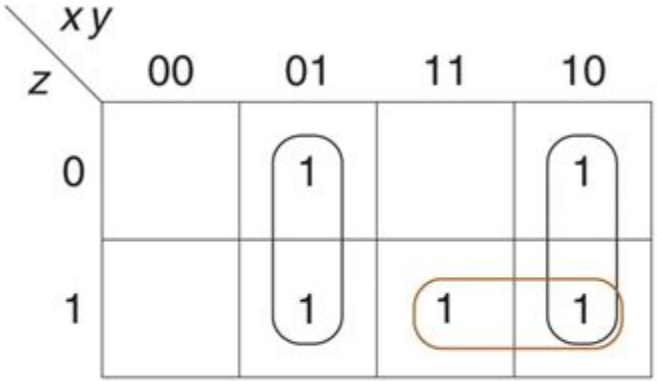
Essential prime implicants are the prime implicants that MUST be used in any min SOP expression

Example

w’x’y’z’ + w’xy’z’ + wxy’z’ + wx’y’z’ + w’xy’z + w’xyz + wxyz + wx’yz

Example

x’yz’ + x’yz + xy’z’ + xy’z + xyz


xy
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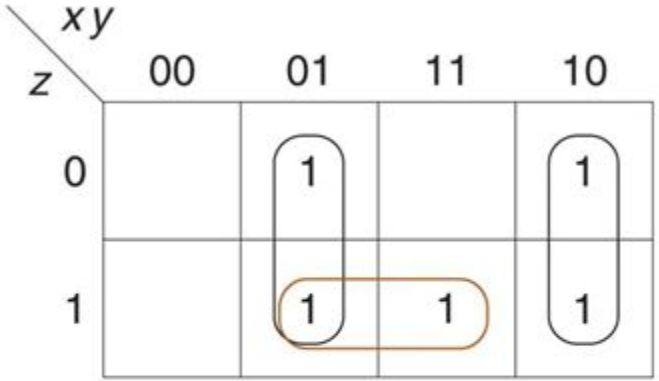
1


xy
z
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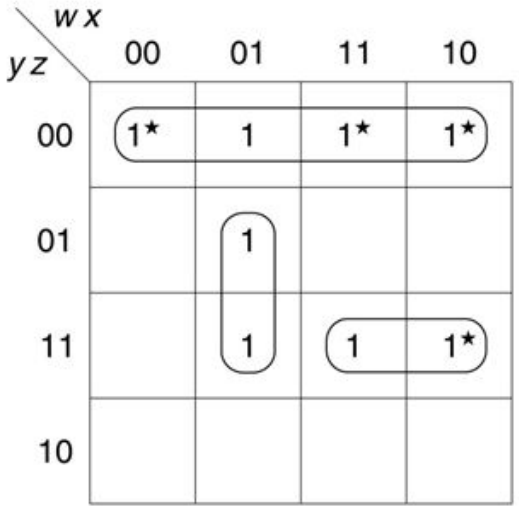
Example

w’xy’z’ + w’xy’z + wxy’z + wx’y’z + w’x’yz + w’xyz + wxyz + wxyz’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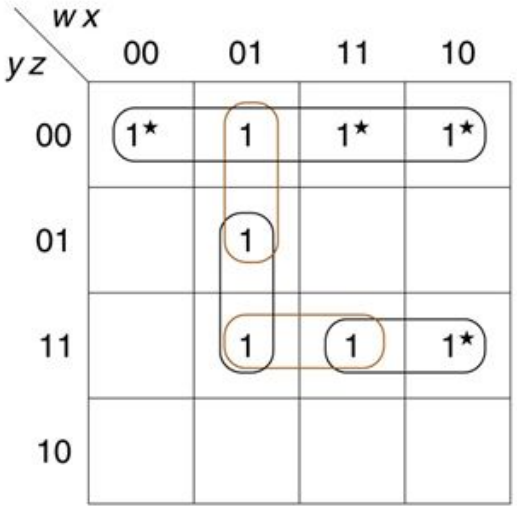
wx
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$$f = y'z' + wyz + w'xz$$

Don’t cares

Implicants can have don’t cares in them too

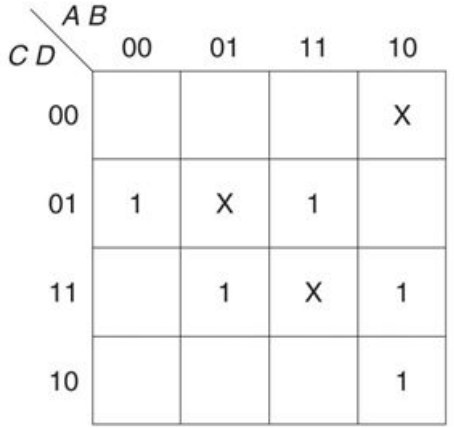
An *implicant* is a rectangle of 1, 2, 4, 8, … 1’s or X’s

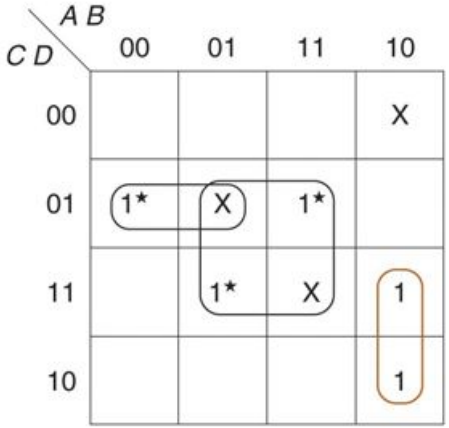
A *prime implicant* is a rectangle of 1, 2, 4, 8, … 1’s or X’s not included in any one larger rectangle. Thus, from the point of view of finding prime implicants, X’s (don’t cares) are treated as 1’s.

An *essential prime implicant* is a prime implicant that covers at least one 1 not covered by any other prime implicant (as always). Don’t cares (X’s) do not make a prime implicant essential.

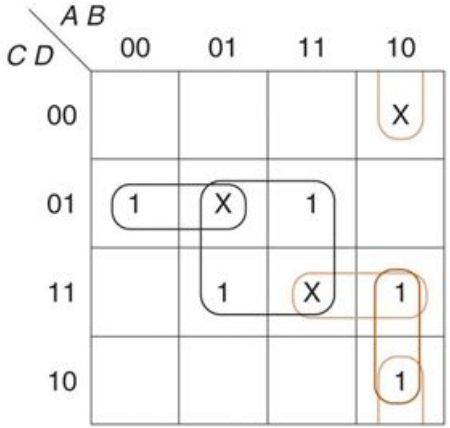
Example

f = SUM[m(1,7,10,11,13)] + SUM[d(5,8,15)]





minimum



other p.i.s

$$F = BD + A'C'D + AB'C$$