

# Lecture 3

or “Two more days until kickoff”

# Announcements

Homework will be assigned at the end of class today. It will be due next Tuesday (Sept. 8th)

# Switching Algebra

- Need algebra to
  - Obtain the output in terms of the input according to the specification of a network of gates
  - Simplify the expression
  - Implement networks of gates

# Operators for Switching Algebra

- OR
  - Also written as +
  - $a + b$  (read as a OR b) is 1 iff  $a = 1$  or  $b = 1$  or both
- Truth table on board

# Operators cont.

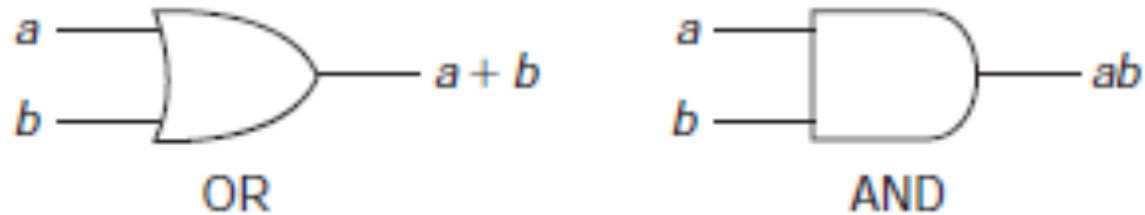
- AND
  - Also written as  $\bullet$  or 2 variables concatenated
  - $a \bullet b = ab$  (read as a AND b) is 1 iff  $a = 1$  and  $b = 1$
- Truth table on board

# Operators cont.

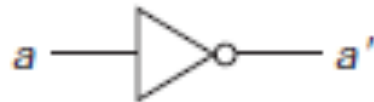
- NOT
  - also written as  $'$
  - $a'$  (read NOT  $a$ ) is 1 iff  $a = 0$
- Truth table on board

# Gate symbols

**Figure 2.4** Symbols for OR and AND gates.



**Figure 2.6** A NOT gate.



# Switching Algebra Properties

- Commutative
  - P1a.  $a + b = b + a$       P1b.  $ab = ba$
- Associative
  - P2a.  $a + (b + c) = (a + b) + c$
  - P2b.  $a(bc) = (ab)c$
- Total of 14 of these...they are in the book



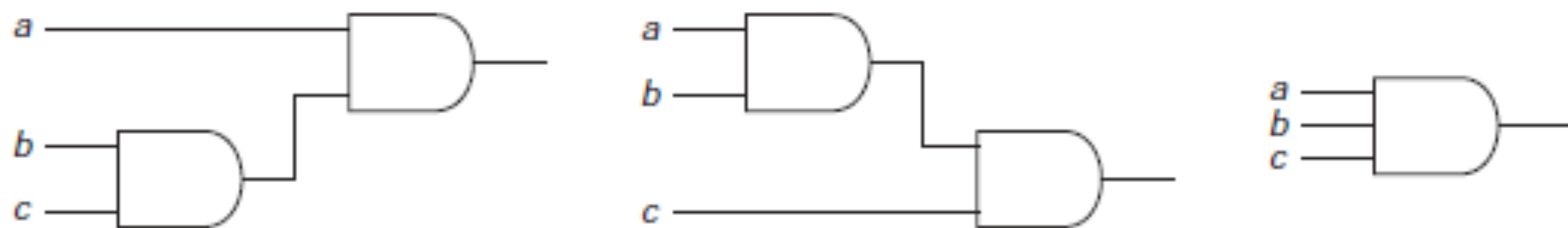
# Generalize

- $a + b + c + d + \dots$  is 1 if any of the operands is 1 and is 0 only if all are 0
- $abcd\dots$  is 1 if all of the operands are 1 and is 0 only if any is 0

# Lets look at P2b

- $a(bc) = (ab)c$
- How would this look with gates?
- How about  $a + (b + c) = (a + b) + c$

**Figure 2.5** AND gate implementation of Property 2b.



# Order of Precedence

- Without parentheses, order of precedence is:
  - NOT
  - AND
  - OR
- Parentheses are always first

# Let's try a few

- $a'b + cd'$ 
  - Assume  $a = 1$ ,  $b = 0$ ,  $c = 1$ , and  $d = 0$

# More properties

- Identity

- P3a.  $a + 0 = a$

- P3b.  $a \cdot 1 = a$

- Null

- P4a.  $a + 1 = 1$

- P4b.  $a \cdot 0 = 0$

- Complement

- P5a.  $a + a' = 1$

- P5b.  $a \cdot a' = 0$

# More!

- Idempotency

- P6a.  $a + a = a$
- P6b.  $a \cdot a = a$

- Involution

- P7.  $(a')' = a$

- Distributive

- P8a.  $a(b + c) = ab + ac$
- P8b.  $a + bc = (a + b)(a + c)$ 
  - Table time

# Definitions of Terms

- Literal: Appearance of a variable or its complement
- Product term: One or more literals connected by AND operators
- Standard product term: Also called minterm, is a product term that includes each variable of the problem, either uncomplemented or complemented
- Sum of products: Also SOP, is one or more terms connected by OR operators



# Cont.

- Canonical sum: (sum of standard product terms) is just a sum of products expression where all of the terms are standard product terms

# More terms

- Minimum sum of products: Expression is one of those SOP expressions for a function that has the fewest number of product terms. If there is more than one expression with the fewest number of terms, then the minimum is defined as one or more of those expressions with the fewest number of literals

# This should be the last one

- Sum term: one or more literals connected by OR operators
- Standard sum term: also called a maxterm, a sum term that includes each variable of the problem, either uncomplemented or complemented
- Product of sums: (POS) one or more sum terms connected by AND operators
- Canonical product: also called product of standard sum terms, is just a product of sums expression where all of the terms are standard sum terms

# The important examples

- Sum of products:
  - $x'y + xy' + xyz$
- Product of sums:
  - $(x + y')(x' + y)(x' + z')$
- Both
  - $x' + y + z$  or  $xyz'$
- Neither
  - $x(w' + yz)$  or  $z' + wx'y + v(xz + w')$

# Examples

- Which of the following equivalent expressions is the minimum SOP expression?
  - $x'y'z' + x'yz' + xy'z' + xy'z + xyz$
  - $x'y + xy' + xyz$
  - $x'y + xy' + xz$
  - $x'y + xy' + yz$

# Homework!

Due next Tuesday

On the website!

# End o' class

- Don't forget homework due on the 8th
- Quick quiz on some of Chapter 2 next class as well