Model Based Control of a Four-Tank System

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Abstract

A multi-disciplinary laboratory for control education has been developed at the University of Delaware to expose students to realistic process system applications and advanced control methods. One of the experiments is level control of a four-tank system. This paper describes two model-based methods students can implement for control of this interacting four-tank system. Sub-space identification is used to develop an empirical state space model of the experimental apparatus. This model is then used for model based control using Internal Model Control (IMC). This represents an application of inner-outer factorization for non-minimum phase multivariable IMC design. Modeling is also performed using step tests and Aspen software for use with Dynamic Matrix Control (DMC).

Keywords: Predictive Control, Internal Model Control, Process Control Education, Experimental Apparatus

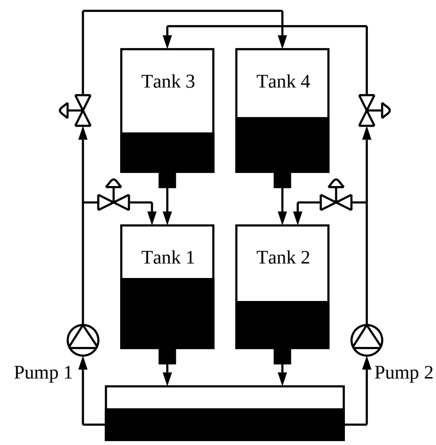

Figure 1: Schematic of the interacting four-tank process. The two manipulated variables are the pump speeds. The two controlled variables are the levels of tanks one and two. Unmeasured flow disturbances can affect tanks three and four.

Introduction

Process control courses for chemical engineers often emphasize complex theoretical and mathematical issues while devoting limited time to implementation of application of control methods. In order to reinforce and demonstrate the concepts presented in a lecture, practical laboratory applications can be developed for students. The experience of working on a laboratory experiment in an academic setting exposes students to realistic industrial problems. Laboratory work can expose students to process details that are often neglected in computer simulation, including measurement noise, measurement bias, process nonlinearity, equipment failure, actuator constraints, and external disturbances.

A multi-disciplinary process control laboratory has been developed at the University of Delaware. Experiments currently include: an inverted pendulum, an electric servo motor, a gyroscope, a distillation column, a spring mass damper system, a virtual boiler, and a four-tank system. The undergraduate experimental control course is offered every year in the Spring semester. Students from mechanical, electrical, and chemical engineering disciplines participate in this class. The students are grouped with students from other majors. This multi-disciplinary group facilitates peer learning in that students familiar with the principles of a given experiment must help other group members. The students enrolled in the process control laboratory have all taken the basic process control course taught in their individual departments. The basic knowledge of process control allows the experimental laboratory course to cover advanced applications of process control.

A four-tank level control system has been constructed for use in the control laboratory. This paper details the use of model identification and model-based control methods for control of this interacting four-tank system. The four-tank system is based on the system presented by Johansson and Nunes [2]. A schematic of the process is shown in

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Figure 1. The experiment has two inputs (pump speeds) which can be manipulated to control the two outputs (tank levels). The system exhibits interacting multivariable dynamics because each of the pumps affects both of the outputs. The system has an adjustable multivariable zero that can be set to a right-half or left-half plane value by changing the valve settings of the experiment. Unmeasured disturbances can be applied by pumping water out of the top tanks and into the lower reservoir. This exposes students to disturbance rejection as well as reference tracking.

$$
\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}\nu_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}\nu_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}\nu_2 - \frac{k_{d_1} d_1}{A_3} \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}\nu_1 - \frac{k_{d_2} d_2}{A_4}
\end{aligned}
$$

Table 1: Nonlinear model equations for the four-tank system.

A full nonlinear mass balance model of the system is given in Table 1. Here, Bernoulli's law is used for flows out of the tanks, h_i is the level of water in tank i, ν_1 and ν_2 are the manipulated inputs (pump speeds), and $d_1$and d_2 are external disturbances representing flow out of tanks three and four. The linearized model is given in Table 2 and the estimated model parameters for the experimental setup are given in Table 3. A_i is the area of Tank i and a_i is the area of the pipe flowing out of tank i. The ratio of water diverted to tank one rather than tank three is γ_1 and γ_2 is the corresponding ratio diverted from tank two to tank four. It can be shown that for the linear system, a multivariable right half plane zero will be present when $\gamma_1 + \gamma_2 < 1$.

$$
\begin{aligned}
\frac{dh}{dt} &= \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} h \\
&+ \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_3} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{k_{d_1}}{A_3} & 0 \\ 0 & -\frac{k_{d_2}}{A_4} \end{bmatrix} d
\end{aligned}
$$

$$
T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i(0)}{g}}
$$

Table 2: Linearized model equations for the four-tank system.

Sub-space identification is used to develop an empirical linear state space model of the experimental apparatus. This model is then used for formulation and application of model-based control methods. The identified state-space

a_1, a_2	$2.3\,cm^2$	k_1	$5.51\,cm^3/s$
a_3, a_4	$2.3\,cm^2$	k_2	$6.58\,cm^3/s$
A_1, A_2, A_3, A_4	$730\,cm^2$	g	$981\,\frac{cm}{s^2}$
$\nu_1(0)$	60%	γ_1	0.333
$\nu_2(0)$	60%	γ_2	0.307
T_1	$53.8\,sec$	$h_1(0)$	$14.1\,cm$
T_2	$48.0\,sec$	$h_2(0)$	$11.2\,cm$
T_3	$38.5\,sec$	$h_3(0)$	$7.2\,cm$
T_4	$31.1\,sec$	$h_4(0)$	$4.7\,cm$

Table 3: Model parameters of the experimental four-tank system.

model is used explicitly in the control algorithms. The methods considered in this work include multivariable Internal Model Control (IMC) and Dynamic Matrix Control (DMC). Non-minimum phase behavior of the system requires that inner-outer factorization be used for the multivariable IMC design. The right-half plane pole also creates performance limitations for the closed-loop system. Because the system has an adjustable zero, the system can be adjusted to exhibit minimum phase behavior and demonstrate improved control response.

The physical system is constructed so that it appears to emulate elements of an industrial unit operation. Four five-gallon tanks are used in the simulation. The addition of submersible pumps in the tanks can simulate a leak disturbance in the tanks. An industrial Distributed Control System (DCS) from ABB-Bailey is used for control of the system using the Freelance software package. The operator interface relies on a PC communicating with the Bailey ProcessStation using TCP/IP on a private LAN. An interface has been developed using Dynamic Data Exchange (DDE) that allows controllers developed using MATLAB and Simulink to be used for simulation runs. The apparatus has the flavor of an industrial system while retaining the flexibility needed to quickly implement and test advanced control strategies.

Sub-Space Process Modeling

Most advanced process control methodologies require the development of accurate models of the system. For identification, the process is typically forced by known inputs and the resulting output responses are used for development of a linear model relating system inputs to outputs. Some process data should be used for validation of new process models. In industrial applications, one must consider tradeoffs between the need for accurate process models, the disadvantage of large excursions from normal operating conditions during modeling, the problem of extended periods of process down time, and the desire for "plant friendly" input sequences.

To effectively balance the process modeling tradeoffs, one may employ a Pseudo-Binary Random Binary Sequence (PRBS) of inputs. This type of input sequence can effectively excite a multivariable process for use in identification and provide an accurate process model while avoiding large process excursions and periods of off specification production. The PRBS of inputs can also be considered a "plant friendly" sequence because only two different levels of input are used.

The resulting input and output data are used to create the linear model. For this task, a sub-space identification procedure is used. This procedure is described in [5]. The method develops an empirical linear state space model from input and output data. For this type of modeling, no prior process knowledge is used; no assumptions are made about the state relationships or number of process states. Only the number of states used in the resulting process model must be determined.

It should be noted again that the sub-space identification method is an empirical method. In many cases, a model based on fundamental process knowledge (first-principles modeling) may be more desirable. This may be true for a process with significant nonlinear characteristics and varied operating regimes. For process that operate around a single steady state or have little nonlinear character, empirical linear modeling can effectively be used for process control.

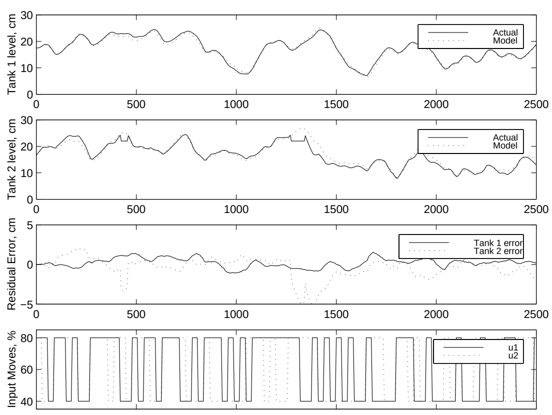

Figure 2: Model validation for sub-space identification on the four-tank system.

Although there are fundamental models available for the four-tank system, the sub-space identification method was used to show that empirical modeling can effectively be applied to real systems. Sub-space identification of a four-tank process has been demonstrated in [2]. For the current study, the normal input levels were 60% for both pumps. A binary sequence was used that switched between levels of 40% and 80%. Switching could occur every 35 seconds. The resulting output levels never exceeded +8 cm or -10 cm in relation to the nominal operating point. The N4SID algorithm from the Matlab Identification Toolbox was used to calculate the linear models. Four states are needed to effectively capture the process character. This result agrees with knowledge of the process and the existing fundamental model. Figure 2 shows a comparison of the process and the model, as well as process residuals.

Internal Model Control

Internal Model Control (IMC) is a very effective method of utilizing a process model for feedback control. IMC directly uses the process model and requires very limited on-line computation. For a full discussion of IMC, see the monograph by Morari and Zafiriou [4]. IMC uses a process model and "inverts" parts of the model for use as a controller for the process. Some portions of a linear process model cannot be inverted. These non-invertible factors include time delays and Right-Half-Plane (RHP) zeros. In addition, a process model that is not semi-proper cannot be inverted. A linear filter can be added to make the process model invertible. The filter parameters then are available for adjusting the aggressiveness of the IMC controller.

The following “inner-outer” factorization for the stable process $G(s)$ follows the procedure described in [4]. The linear process transfer function can be written:

$$G(s) = C(sI - A)^{-1}B + D = N(s)M(s)^{-1}$$

where $N(s)$ and $M(s)$ are stable. Additionally, $N(i\omega)^H N(i\omega) = I$. The non-invertible portion of the process, $N(s)$, is given by:

$$N(s) = (C - QF)(sI - (A - BR^{-1}F))^{-1}BR^{-1} + Q$$

For $N(s)$, the state space matrices are given as:

$$\begin{array}{ll} A_N = A - BR^{-1}F & B_N = BR^{-1} \\ C_N = C - QF & D_N = Q \end{array}$$

The invertible part of the process, $M(s)^{-1}$, is:

$$M(s)^{-1} = F(sI - A)^{-1}B + R$$

For $M(s)$, the state space matrices are given as:

$$\begin{array}{ll} A_M = A & B_M = B \\ C_M = F & D_M = R \end{array}$$

The following relation is given for R and Q:

$$D = QR$$

For this example, the orthogonal matrix Q is selected as identity. To make the process semi-proper, D is set to ϵI, with $\epsilon = 0.0001$. For the factorization method, F is calculated as

$$F = Q^T C + (BR^{-1})^T X$$

where X is the solution of the algebraic Riccati equation:

$$0 = (A - BR^{-1}Q^T C)^T X + X(A - BR^{-1}Q^T C) \\ -X(BR^{-1})(BR^{-1})^T X$$

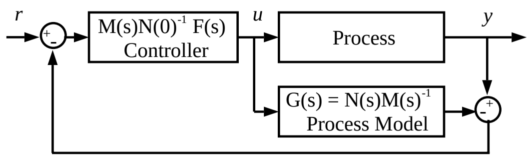

Figure 3: Internal Model Control (IMC) block diagram showing “Inner-Outer” factorization of plant $G(s)$ into the invertible portion $M(s)^{-1}$ and the non-invertible portion $N(s)$. $F(s)$ is a filter used to adjust the aggressiveness of the control loop.

The controller for the IMC formulation is the inverse of the invertible portion of the process model. For offset free steady state reference tracking in all channels, the product of the controller inverse gain and the process model gain must be identity. The current process model factorization does not guarantee this. It can conveniently be achieved by scaling the non-invertible portion of the plant by $N(0)^{-1}$. Now, the noninvertible process model is $N(s)N(0)^{-1}$ and the invertible process model is $N(0)M^{-1}(s)$. The IMC controller becomes $M(s)N(0)^{-1}$. SISO IMC systems incorporate a scalar filter for strictly-proper process models so that the resulting controller is semi-proper. In this multivariable case, the process model is already semi-proper. Each of the error signals sent to the MIMO 2x2 IMC controller can be filtered with a first-order linear filter, $F_i(s) = \frac{1}{\lambda_i s+1}$. This filter allows for adjustment of the closed-loop dynamics.

Figure 4 shows closed-loop performance of the IMC system for reference changes. In this example, λ_i are selected to 7 sec.

The process has zeros at values of 0.0324 and -0.088. The input direction corresponding to the RHP (as described in [6]) is: $[-0.736 \;\; 0.677]^T$ and the output direction is $[-0.693 \;\; 0.721]^T$. Multivariable performance limitations related to RHP systems are described in [3]. In the nominal case, the process is identical to the process model and the filter time constants are set to 0. The complementary sensitivity function, $T(s)$, for the nominal IMC system reduces to $N(s)N(0)^{-1}$. Ideally, $T(s)$ is identity at all frequencies. The presence of a RHP zero creates a performance limitation for the system. In [4], it is shown that $T(s)$ for a nonminimum phase system with a single zero can be arbitrarily selected so that only a single row deviates from identity. This implies that the performance degradation caused by the RHP zero can be driven into a single output channel. The complementary sensitivity function for the 2x2 tank system with ideal performance in the first measurement is:

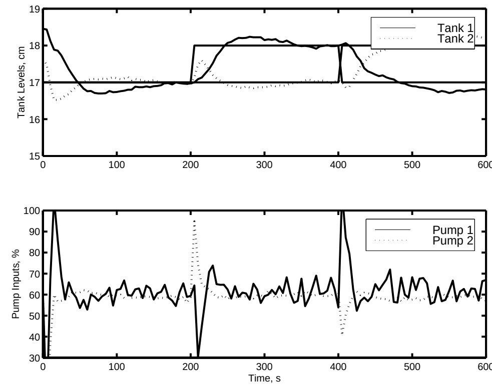

Figure 4: Closed-loop reference tracking using IMC controller with $\lambda = 7sec$.

$$T_1(s) = \begin{bmatrix} 1 & 0 \\ \frac{\beta_1 s}{s+\zeta} & \frac{-s+\zeta}{s+\zeta} \end{bmatrix}$$

and the complementary sensitivity function for the system with ideal performance in the second measurement is:

$$T_2(s) = \begin{bmatrix} \frac{-s+\zeta}{s+\zeta} & \frac{\beta_2 s}{s+\zeta} \\ 0 & 1 \end{bmatrix}$$

where β_1 and β_2 are functions of the terms in the output zero direction. Significant interaction can occur when the values of β_i are large. For the given system, β_1 is 1.9 and β_2 is 2.1. This implies that choosing either input for ideal tracking in the nominal case will have essentially the same amount of interaction. This result is expected due to the symmetric nature of the system. For the developed controller formulation, a first-order realization of $T(s)$ is given as:

$$T(s) = \begin{bmatrix} \frac{0.039s+1}{s+0.32} & \frac{1s+1}{s+0.32} \\ \frac{1s}{s+0.32} & \frac{-0.039s+1}{s+0.32} \end{bmatrix} = \begin{bmatrix} \frac{1.2s+1}{32s+1} & \frac{32s}{32s+1} \\ \frac{32s}{32s+1} & \frac{-1.2s+1}{32s+1} \end{bmatrix}$$

This also demonstrates that the system and controller formulation is symmetric. Both output channels should demonstrate equal performance limitations and interactions.

Dynamic Matrix Control

Dynamic Matrix Control (DMC) refers to the commercial implementation of model predictive control (MPC) provided by Aspen Technology. Aspen is a member of the University of Delaware Process Monitoring and Control Consortium and has provided DMCPlus for student use in University of Delaware teaching laboratories.

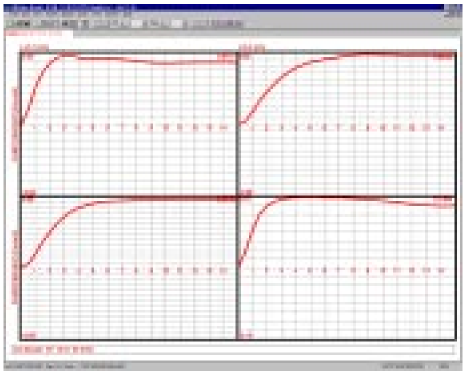

Figure 5: Screen shot of the DMC four-tank system modeling interface.

Like other MPC algorithms, DMC uses an optimization criterion to choose control moves. Using an explicit process model for prediction of process outputs, DMC chooses an optimal sequences of control moves based on a trade-off between speed of setpoint tracking and avoidance of rapid changes in control inputs. Some specific features of the DMC algorithm include the following:

- Applicability to multivariable and non-square systems
- A step response model
- A quadratic optimization criterion
- Explicit tradeoff between setpoint tracking and aggressive of control action through an "input suppression factor"
- Continuous adjustment of setpoints via linear programming to target the most profitable steady-state operating condition

DMC has been presented for control of a similar process [1]. In the DMCPlus implementation presented here, students in the control teaching laboratory learn an advanced multivariable control package that is not only industrially relevant, but is *identical* to that found in industry. Figure 5 shows the DMCPlus modeling interface. Figure 6 shows a screen shot of the DMCPlus interface to the four-tank system. The DMCPlus system permits students to explore several interesting aspects of the four-tank system:

- Controller Tuning: Aggressive tuning of a DMC or other MPC controller has the effect of computing a controller that is the inverse of the process. Since the four-tank system can be adjusted to have a right half-plane zero, aggressive controller tuning can lead to a controller that has a right half-plane pole and is therefore unstable. Investigations on the limits of controller tuning and ability to track setpoint changes offer interesting opportunities for student experiments.
- Process Identification: One of the first steps in implementing a DMC controller is the identification of a step response model for the process. The associated software tools can be used to store and implement different process models for different operating conditions. In this way, students can also investigate the effects of model mismatch on closed-loop performance
- Gain scheduled DMC: DMCPlus contains features to permit process gains to vary with operating conditions, thereby incorporating a particular form of nonlinear model into the DMC algorithm. In the four-tank system, this permits models to be used over a wider operating range, and students can compare the differences between gain-scheduled DMC to single-model DMC.

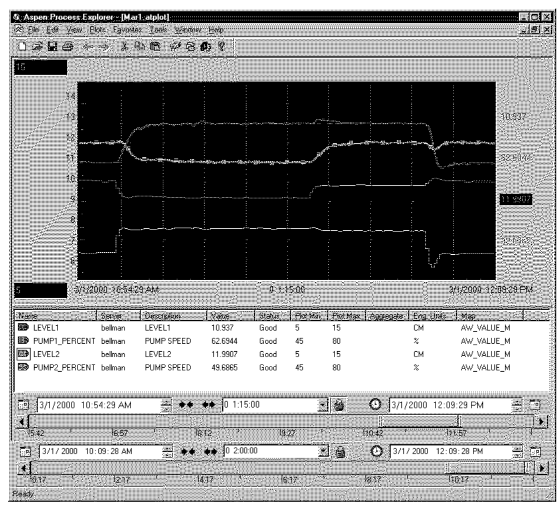

Figure 6: Screen shot of the four-tank system DMC interface showing a closed-loop run.

Implementation Details DMC implementation on the four-tank system has some significant differences from typical industrial implementations:

- An industrial implementation of DMC is implemented in a cascade configuration that is built upon a system of well-tuned, single-input/single-output

(SISO) regulatory controllers. Industrial DMC then chooses setpoints to be implemented in these regulatory controllers to effect process outputs. With the simpler experimental process, it is not possible to implement DMC in a cascade formulation since the necessary instruments to measure flow are not present. Therefore, the DMC controller directly manipulates pump speed to achieve optimal dynamic performance.

- Industrial implementation of DMC depend upon economic criteria to determine steady state setpoints, based upon prices of input and output streams in a process. Since the laboratory system has no economic criteria assigned to its operations, we have to substitute equivalent criteria as DMC tuning parameters. In these experiments, we assigned a negative value (a cost) to pump speed and asked for the DMC controller to choose operating points that would maximize profits (minimize costs) subject to a minimum-level constraint on measured tank levels. This is equivalent to achieving a setpoint for tank levels with minimum steady-state control action.

Conclusions

This paper has described advanced modeling and control techniques that can be used by students in a hands-on experimental control laboratory. The students will be introduced to sub-space identification for plant-friendly process identification. IMC is applied to the nonminimum phase process by mathematically factoring the model into invertible and non-invertible sections. DMC is used to control the system and explicitly implement process constraints. Students are introduced to advanced modeling and control techniques in an application based environment. They are able to connect classroom theory with concrete laboratory experiences.

The first offering of the laboratory class used the four-tank system for SISO and multivariable decoupling. The advanced identification and control methods presented in this paper are being developed for use in upcoming course offerings. The 15 students from the first offering of the laboratory class were asked to evaluate the laboratory course. These students were asked to rate the overall educational value as well as the practical value of each laboratory experiment on a scale of 1 to 5, 5 being the highest. Table 4 shows the quantitative ratings of the separate experiments used in the first offering of the course. Note that the four-tank system received the highest rating of the five experiments. The students were also asked to give a free response evaluation of each experiment. Some representative student quotes from the free response section of the four-tank evaluation include:

Very good lab, extremely educational.

This was the most intuitive control problem for me, very easy to see the direct physical results of our control action. My personal favorite.

Real problem using industrial interfaces. Excellent practical problem, but it takes too long due to the time constant.

	Q1 Avg.	Q1 St. Dev.	Q2 Avg.	Q2 St. Dev.
Four-Tank	4.0	0.93	4.4	0.91
Spring Mass	3.5	0.92	3.3	1.1
Pendulum	2.9	1.3	3.2	1.1
Servo Motor	2.9	0.99	3.2	1.1
Virtual Boiler	2.4	1.2	3.1	1.3

Table 4: Student evaluations of the multi-disciplinary laboratory experiments. There were 15 students taking the class. Question 1 was: *"Rate the overall educational value of each experiment using a scale of 1-5 (1=poor, 5= excellent)."* Question 2 was: *"Rate the practical value of each experiment using a scale of 1-5 (1=poor, 5= excellent)."*

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