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# ADVANCED PROCESS CONTROL

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This book is dedicated to my inquisitive students,  
who surely have been my best teachers.

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## PREFACE

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This book is designed to be used as a text for advanced undergraduate and graduate courses in process control as well as to serve as a reference for the practicing control engineer. Bearing in mind the likely background of the reader, the mathematical prerequisites consist of introductory undergraduate process control material, elementary matrix algebra, and an introduction to differential equations. The standard undergraduate chemical engineering curriculum usually provides the necessary background. In spite of these modest mathematical demands, the book strives to provide a broad coverage of applied modern control theory. Although only one or two sections may be devoted to theoretical subjects upon which entire books have been written, the goal is to provide the process control engineer a synopsis of the theory, a clear exposition of the most useful design techniques, and example problems to demonstrate the key features of the computational algorithms. Numerous references are provided so that the interested reader may delve more deeply into the mathematical theory or seek further illustrative examples.

The material has been tested in the classroom over the last five years both at the State University of New York at Buffalo and at the University of Wisconsin. While the bulk of the students have been undergraduate and graduate students in chemical engineering, these classes have also included both electrical and mechanical engineering students interested in process control. Usually these courses have involved laboratory projects through which students are able to implement some of their control system designs.

It is perhaps useful to point out the thrust of each chapter. In Chapter 1 there is a discussion of the engineering and economic incentives for implementation of advanced process control concepts. The key elements of an advanced control scheme are discussed and their integration into a coherent, comprehensive structure is outlined. This is followed by an overview of the different types of process dynamics models normally encountered. This provides a guide



for the ensuing chapters, which treat control system design for each type of process model.

Chapter 2 is an introduction to the computer and interfacing technology needed for real-time process control. The basic structure of microcomputers and minicomputers is discussed together with the features of peripheral devices. Both digital and analog data acquisition interfaces are covered, and their essential features are discussed. A substantial number of process sensors, control actuators, and transducers are described in the context of their typical applications. Finally, several actual process control interfacing projects are presented and the ultimate design strategy with regard to sensor selection, multiplexing, signal conditioning, mode of transmission, etc., is discussed. This chapter is designed to provide the reader a good grasp of the considerations necessary to implement computer control schemes. The intention is to give the needed perspective and motivation for the more theoretical material to follow.

Chapter 3 deals with the problem of control system synthesis for lumped parameter systems. Both differential equation state space models and transfer function models are considered, and the equivalence of the two formulations is demonstrated. Standard design procedures for linear multivariable systems are introduced to demonstrate that these "suboptimal" procedures often produce high quality, but simple, designs. Then classical optimal control theory is presented, both to provide a basis for comparison with simpler suboptimal strategies and to lay a theoretical foundation which can be built upon in later discussions of state estimation and stochastic control. Finally, several methods of nonlinear system controller design are presented.

Chapter 4 introduces the reader to distributed parameter systems which are modeled by partial differential or differential-delay equations. As in the previous chapter, there is a discussion of the types of distributed systems commonly encountered in the process industries and the appropriate techniques for control system design. Following this, optimal control theory is presented and optimal controller design illustrated by examples. Where applicable, modal decomposition for linear systems and Galerkin procedures for nonlinear systems are stressed. The chapter ends with a treatment of systems having time delays. The goal of this chapter is to demonstrate that through appropriate analysis, distributed systems are not significantly more difficult to handle than lumped systems.

Chapter 5 is devoted to state estimation and stochastic control. Although the relevant statistical theory can get rather deep at times, particularly for nonlinear systems, the essential features are presented in an operational-intuitive manner suitable for the reader with only a rudimentary background in statistics. This approach is designed to allow the control engineer to see the thrust of the analysis and understand the important results so that useful estimation and control algorithms may be developed.

Chapter 6 consists of a series of detailed case studies of control system design. For each control problem treated, the reader is led through the steps of control system synthesis and then shown the performance of the resulting design when applied to a pilot plant process. It is thought that these exercises in



*synthesis*, which draw upon all the techniques covered in the earlier chapters, will demonstrate the power of these sophisticated and useful process control methods in a practical context.

There are many who contributed their efforts to this book. Several generations of students provided example problems, criticism, and testing of the homework assignments; Klavs Jensen and Tunde Ogunnaike were particularly helpful in carrying out computations for example problems. Proofreading help was provided by Klavs Jensen, Tunde Ogunnaike, Joe Schork, and Marvin Schwedock, while Alan Schmidt kindly served as photographer for some of the half-tones. Professors Jay Bailey, Ram Lavie, Manfred Morari, John Seinfeld, and Joe Wright read the manuscript and contributed many helpful suggestions. The detailed comments resulting from the classroom testing of the material by my colleague, Manfred Morari, were especially valuable. Finally there were the typing efforts of Diane Petersen and my wife, Nell. Their skill transformed barely legible scribbling into flawlessly typewritten copy, while their patience allowed numerous revisions of the original manuscript. As usual, this project could not have been completed without the devotion of my wife, Nell, who participated in all aspects of the book (typing, preparation of the index, proofreading, etc.) while keeping up with all her other activities. The author is greatly indebted to all who contributed to this venture.

*W. Harmon Ray*

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## ADVANCED PROCESS CONTROL

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## INTRODUCTION

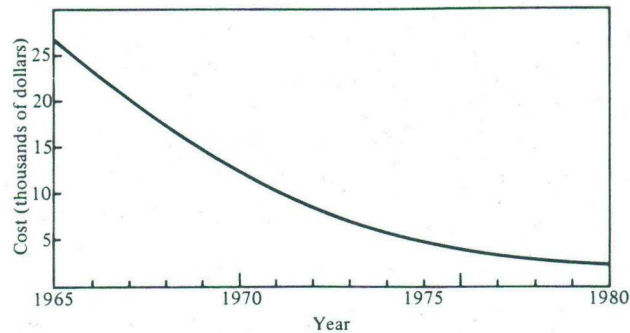
### 1.1 WHY ADVANCED PROCESS CONTROL

From its beginnings in antiquity [1, 2] until the early 1960s, the field of process control was based almost entirely on mechanical, electrical, or pneumatic analog controllers, which were usually designed using linear single-input single-output considerations. Hardware limitations, economic cost, and the dearth of applicable theory usually precluded anything more complex than these simple schemes. Because many large-scale industrial processes are endowed by nature with large time constants, open-loop stability, and significant damping of fluctuations through mixing and storage tanks, such simple control schemes work well for perhaps 80 percent of the control loops one might encounter. For the remaining 20 percent more difficult control problems, most controllers were considered marginally acceptable during this early period because there were few environmental regulations, product specifications were quite loose, and intermediate blending tanks could cover many of the sins of inadequate control. Thus the costs of even small sophistications in control were high and the economic incentives for improved control were comparatively low.

Over the last 10 to 15 years, there has been a dramatic change in these factors. Industrial processes are now predominantly continuous with large throughputs, highly integrated with respect to energy and material flows, constrained very tightly by high-performance process specifications, and under intense governmental safety and environmental emission regulations. All these



## 2 ADVANCED PROCESS CONTROL



**Figure 1.1** An example of price trends for real time minicomputers.

factors combine to produce more difficult process control problems as well as the requirement for better controller performance. Significant time periods with off-specification product, excessive environmental emissions, or process shut-down due to control system failure can have catastrophic economic consequences because of the enormous economic multipliers characteristic of high-throughput continuous processes. This produces large economic incentives for reliable, high-quality control systems in modern industrial plants.

Another recent development in process control is that the performance of real-time digital computers suitable for on-line control has improved significantly, while prices have fallen drastically. Figure 1.1 shows an example of the price trends for small minicomputers in spite of the inclusion of more reliable electronics and increasing inflation. With the process control computer now such a small part of the overall process capital costs, the installation of a fast minicomputer with large amounts of storage can often be easily justified on the basis of improved safety and manpower savings. Once in place, the computer is usually operating in a timesharing mode with large numbers of input/output operations, so that the central processing unit (CPU) is typically in use only about 5 percent of the time. Thus many installations have 95 percent of the computing power of a highly capable minicomputer, programmable in a high-level language such as Fortran, already available for implementing sophisticated computer control schemes.

At the same time, modern control theory has been under intense development, with many successful applications in the aerospace and aircraft industries. Most recently, a number of process control research groups have been applying these ideas to simulated, laboratory, and even a few full-scale plant processes, so that much of the applicable theory needed for sophisticated process control algorithms is at hand. However, there remains the problem of communicating these results, in a readily applicable form, to the process control engineer who must design an economically optimal process control scheme. The present text is directed toward this educational need, and strives to present a comprehensive introduction to the theory and practice of modern computer process control.



## 1.2 WHAT IS "MODERN" CONTROL THEORY?

In contrast to classical control theory, which is essentially limited to single-input single-output systems described by linear differential equations with constant coefficients (or their corresponding Laplace transforms), so-called *modern* control theory has developed to the point where results are available for a wide range of general multivariable systems, including those described by:

1. Linear, variable-coefficient differential equations
2. Nonlinear differential equations
3. Differential-difference and other hereditary equations
4. Partial differential and integral equations

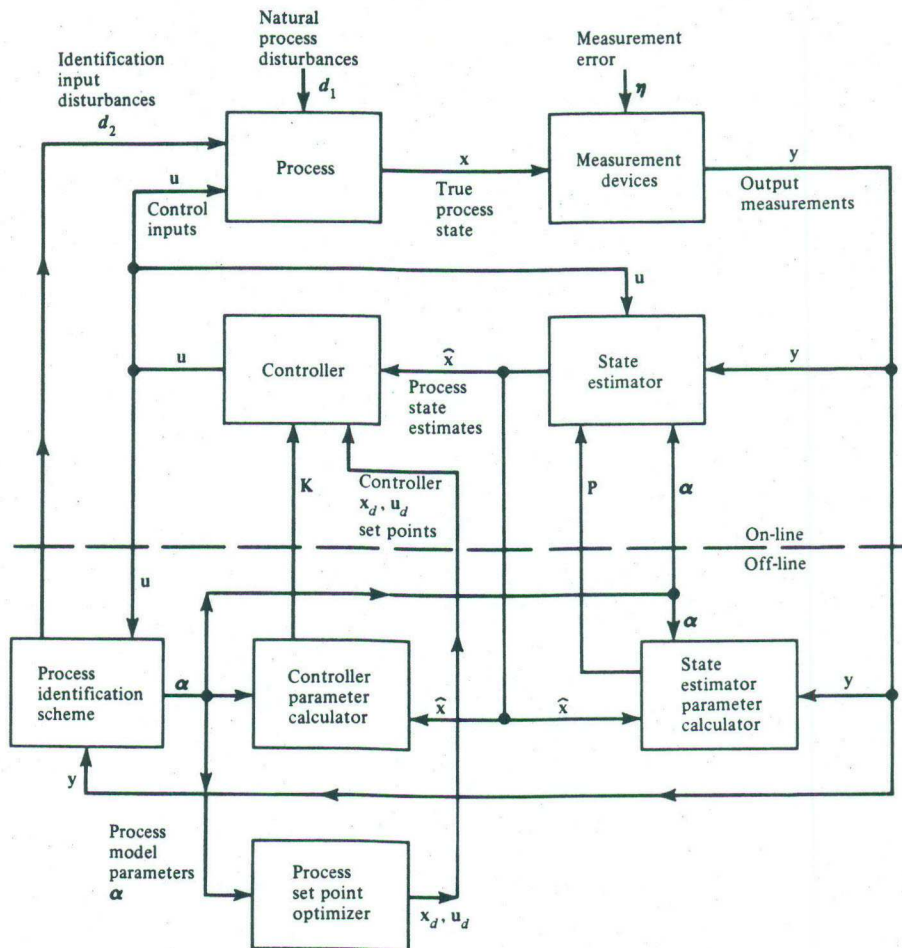
The results of modern control theory include so-called *optimal control theory*, which allows the design of control schemes which are *optimal* in the sense that the controller performance minimizes some specified cost functional.

In addition to controller design, modern control theory includes methods for process identification and state estimation. *Process identification* algorithms have been developed which determine the model structure and estimate the model parameters, either off-line or adaptively on-line. These are useful both in the initial control system design and in the design of *adaptive control systems* which respond to such changes in the process characteristics as might arise, for example, with the fouling of heating exchanger surfaces or the deactivation of catalyst in chemical reactors. *State estimation* techniques are on-line methods of estimating those system state variables which are not measured and for improving the quality of all the state-variable estimates when there are measurement errors. In processes where numerous sensors are not available or are expensive, on-line state estimation can be of significant practical importance.

It is perhaps useful to demonstrate how all these components of a comprehensive computer process control scheme might fit together for a particular process and to point to where the technical details are discussed in the text. Figure 1.2 outlines such a control scheme which consists of the following:

1. *Process* which responds to control inputs  $u$ ; natural process disturbances  $d_1$ ; and special input disturbances  $d_2$  which enhance identification. The true process state  $x$  is produced, but this is seldom measured completely or without error so that one employs
2. *Measurement devices*, which usually are able to measure only a few of the states or some combination of states and are always subject to measurement error. These are discussed in Chap. 2. The measurement device outputs  $y$  are fed to the
3. *State estimator*, which makes use of the noisy measurements  $y$  along with a process model to reconstruct the best possible process state estimates  $\hat{x}$ . The state estimator must have parameters  $P$  which are calculated (either off-line

#### 4 ADVANCED PROCESS CONTROL



**Figure 1.2** A comprehensive computer control scheme.

or periodically updated on-line) by algorithms, possibly requiring the measurement outputs  $y$  and state estimates  $\hat{x}$  as well as the process model parameters  $\alpha$ . The details of state estimation are found in Chap. 5. The process state estimates are passed to the

4. *Controller*, which calculates what control actions must be taken based on the state estimates  $\hat{x}$ , the set points  $u_d, x_d$  (which themselves may be the subject of process optimization [3]), and the controller parameters  $K$ . The controller parameters  $K$  can be calculated either off-line or adaptively on-line based on current estimates of the model parameters  $\alpha$  and process state  $\hat{x}$ . Techniques for controller designs are discussed in Chap. 3 and 4. The process model parameters must be determined from the



5. *Process identification scheme*, which takes measurements from the process as raw data  $y$  (and may choose to introduce experimentally designed input disturbances  $d_2$ ) in order to identify the process model parameters  $\alpha$ . If the parameters are invariant with time, then this need be done only once; however, if the process changes with time, then the process identification scheme may be activated periodically to provide adaptation to changing conditions. Process identification is not treated in great detail in this book because there are many fine existing reference books in this area [4–10].

In most applications only a few of the components of this control structure are required. Chapter 6 describes some control system design case histories which illustrate this point.

### 1.3 MATHEMATICAL MODELS FOR PROCESS DYNAMICS

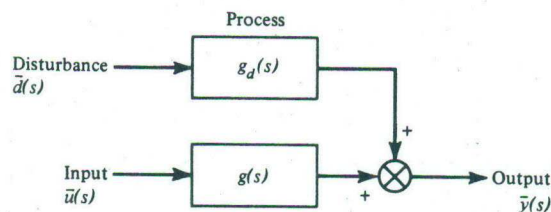
For all but the simplest control schemes to be effective, some description of the process to be controlled must be available. Usually this description is a *mathematical model*. In classical single-loop process control theory, this model often takes the form

$$\bar{y}(s) = g(s)\bar{u}(s) + \bar{g}_d(s)\bar{d}(s) \quad (1.3.1)$$

where  $\bar{d}(s)$ ,  $\bar{u}(s)$ ,  $\bar{y}(s)$  are the Laplace transforms of known disturbances  $d(t)$ , the controller input  $u(t)$ , and process output  $y(t)$ , respectively. Here  $g(s)$ ,  $\bar{g}_d(s)$  are the scalar transfer functions relating them as shown in Fig. 1.3. However, many processes are much more complicated and require more complex models. We shall discuss some of the naturally occurring models in what follows.

#### Linear Multivariable, “Lumped” Models

“Lumped parameter” or “lumped” models refer to process descriptions in which there is no spatial dependence; either *ordinary differential equation* or *Laplace transform* models are employed. The simplest case of a lumped multivariable process model would be a generalization of the single-input single-output transfer function model [Eq. (1.3.1)] to multiple inputs and multiple outputs, as shown in Fig. 1.4. In this case there are  $k$  disturbances,  $m$  inputs, and  $l$  outputs



**Figure 1.3** Single-input single-output linear system.

## 6 ADVANCED PROCESS CONTROL

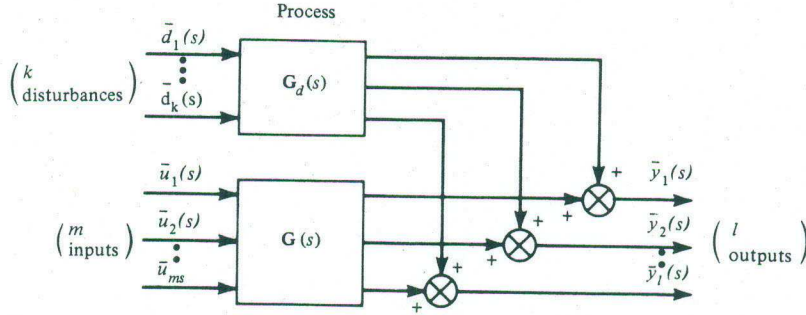


Figure 1.4 Multivariable linear system with  $k$  disturbances,  $m$  inputs, and  $l$  outputs.

related by

$$\bar{y}(s) = G(s)\bar{u}(s) + G_d(s)\bar{d}(s) \quad (1.3.2)$$

where  $\bar{d}(s)$ ,  $\bar{u}(s)$ ,  $\bar{y}(s)$  are vectors and  $G(s)$  is an  $l \times m$  matrix, while  $G_d(s)$  is an  $l \times k$  matrix.

$$\bar{d}(s) = \begin{bmatrix} \bar{d}_1(s) \\ \bar{d}_2(s) \\ \vdots \\ \bar{d}_k(s) \end{bmatrix} \quad \bar{u}(s) = \begin{bmatrix} \bar{u}_1(s) \\ \bar{u}_2(s) \\ \vdots \\ \bar{u}_m(s) \end{bmatrix} \quad \bar{y}(s) = \begin{bmatrix} \bar{y}_1(s) \\ \bar{y}_2(s) \\ \vdots \\ \bar{y}_l(s) \end{bmatrix} \quad (1.3.3)$$

$$G_d(s) = \begin{bmatrix} g_{11_d}(s) & g_{12_d}(s) & \cdots & g_{1k_d}(s) \\ g_{21_d}(s) & g_{22_d}(s) & \cdots & g_{2k_d}(s) \\ \cdots & \cdots & \cdots & \cdots \\ g_{l1_d}(s) & & & g_{lk_d}(s) \end{bmatrix}$$

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1m}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2m}(s) \\ \cdots & \cdots & \cdots & \cdots \\ g_{l1}(s) & g_{l2}(s) & & g_{lm}(s) \end{bmatrix}$$

An alternative form of linear, lumped parameter models is the *time-domain* model:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) + \Gamma d(t) \quad x(0) = x_0 \quad (1.3.4)$$

$$y = Cx(t) \quad (1.3.5)$$

where the variables  $x_i(t)$ ,  $i = 1, 2, \dots, n$ , are the *state variables*; the  $y_j(t)$ ,  $j = 1, 2, \dots, l$ , are the *output variables*; the  $u_r(t)$ ,  $r = 1, 2, \dots, m$ , are the *input variables* or *control variables*; and the  $d_s(t)$ ,  $s = 1, 2, \dots, k$ , are the *disturbance variables*.



The form of the *transform-domain* model [Eq. (1.3.2)] or *time-domain* model [Eqs. (1.3.4), (1.3.5)] may be chosen by convenience because they may be shown to be equivalent if **A**, **B**, **C**, and **D** are constant matrices. The basis of these multivariable models may be determined from the fundamental differential equations of physics or, more often, from empirical fitting of the model equations to measured process dynamics. In either case, there is a wide range of processes which may be approximately modeled in this form. More will be said about these models in Chap. 3.

### Nonlinear Lumped Models

Systems described by nonlinear ordinary differential equations are very common in the process industries and have models of the general form

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{d}(t)) \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (1.3.6)$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t)) \quad (1.3.7)$$

where as before  $\mathbf{x}(t)$  represents a vector of states,  $\mathbf{y}(t)$  the outputs,  $\mathbf{d}(t)$  the disturbances, and  $\mathbf{u}(t)$  the control inputs. The nonlinear functions  $\mathbf{f}$  and  $\mathbf{h}$  may have a variety of forms and are usually derived from the momentum, energy, and material balances and the type of sensors used for the process under study. As an example, a liquid-phase reaction,  $A \rightarrow B$ , carried out in a continuous stirred tank reactor will have a nonlinear model of the form

$$\frac{dx_1}{dt} = -x_1 + \text{Da}(1 - x_1)\exp\left(\frac{x_2}{1 + x_2/\gamma}\right) + d_1 \quad (1.3.8)$$

$$\frac{dx_2}{dt} = -(1 + \beta)x_2 + B\text{Da}(1 - x_1)\exp\left(\frac{x_2}{1 + x_2/\gamma}\right) + \beta u + d_2 \quad (1.3.9)$$

where  $x_1$  is the conversion of  $A$  to product and  $x_2$  is a dimensionless reactor temperature. The quantities  $\text{Da}$ ,  $\gamma$ ,  $B$ , and  $\beta$  are parameters, and  $u$  is the jacket coolant temperature which is manipulated to achieve temperature control. The variables  $d_1$  and  $d_2$  are disturbances in the feed reactant concentration and feed temperature, respectively. The output equation is

$$y = x_2 \quad (1.3.10)$$

which indicates that only reactor temperature is measured. The control of this class of processes is discussed in Chap. 3.

### Distributed Parameter Models

A significant number of industrial processes are “distributed” in space so that their behavior depends on spatial position as well as time. These models usually take the form of partial differential equations and are often derived from the

fundamental balances of momentum, energy, and material for the process at hand. As an example, many heat conduction, fluid flow, and chemical reactor processes take the form

$$\frac{\partial \mathbf{x}}{\partial t} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \nabla \cdot \mathbf{x}, \nabla^2 \mathbf{x}, \dots) \quad (1.3.11)$$

where both the state variables  $\mathbf{x}$  and control variables  $\mathbf{u}$  may depend on time and on a multidimensional spatial position. A simple illustration of this type of distributed model may be found by considering that the liquid-phase reaction  $A \rightarrow B$  noted above is now carried out in a tubular reactor with cooling at the wall. A reasonable model of such a reactor is in the form

$$\begin{aligned} \frac{\partial x_1(r, z, t)}{\partial t} = & (r^2 - 1) \frac{\partial x_1}{\partial z} + \frac{1}{\text{Pe}_m} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial x_1}{\partial r} \right) \\ & + \text{Da}(1 - x_1) \exp\left(\frac{x_2}{1 + x_2/\gamma}\right) \end{aligned} \quad (1.3.12)$$

$$\begin{aligned} \frac{\partial x_2(r, z, t)}{\partial t} = & (r^2 - 1) \frac{\partial x_2}{\partial z} + \frac{1}{\text{Pe}_h} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial x_2}{\partial r} \right) \\ & + B \text{Da}(1 - x_1) \exp\left(\frac{x_2}{1 + x_2/\gamma}\right) \end{aligned} \quad (1.3.13)$$

with boundary conditions

$$\begin{aligned} z = 0: \quad & x_1 = x_2 = 0 \\ r = 0: \quad & \frac{\partial x_1}{\partial r} = \frac{\partial x_2}{\partial r} = 0 \\ r = 1: \quad & \frac{\partial x_1}{\partial r} = 0 \quad \frac{\partial x_2}{\partial r} = \text{Bi}_w(u - x_2) \end{aligned} \quad (1.3.14)$$

Here  $x_1$  and  $x_2$  are reactor conversion and temperature, respectively, and  $u$  is the wall coolant temperature which can be manipulated. The quantities  $\text{Pe}_m$ ,  $\text{Pe}_h$ ,  $\text{Da}$ ,  $B$ ,  $\beta$ ,  $\gamma$ , and  $\text{Bi}_w$  are parameters, and the output equations are of the form

$$y_i(t) = x_2(r_i, z_i, t) \quad i = 1, 2, \dots, N \quad (1.3.15)$$

and represent the thermocouples placed at positions  $(r_i, z_i)$  in the reactor. Note that the control variable in this example problem appears in the boundary condition, Eqs. (1.3.14), as often happens in distributed parameter control problems. Chapter 4 will treat the control of this class of processes in some detail.

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