After an embarrassing karaoke performance at the annual Christmas party, your boss moved you to take charge of the plant human sewage treatment plant at your facility.

1. Draw a process flow diagram for the system from the following information:
   
   (a) Raw sewage enters a mixing / aeration tank
   (b) The mixing tank empties into a holding pond
   (c) Chlorine is added in the holding pond to further reduce the concentration
   (d) The holding pond flows down a long pipe to the river
   (e) Some of the holding pond sewage is recycled to the mixing / aeration tank
   (f) You can adjust the speed of mixing (this affects the mixing tank removal rate)
   (g) You can adjust the rate of chlorine addition
   (h) The feed raw sewage is diluted with clean water and mixed in a pipe before it flows to the mixing / aeration tank
   (i) You can adjust the flow rate of dilution water
   (j) You can measure the concentration of sewage in the feed stream. This may vary with time, especially on “Burrito Friday”
   (k) You can measure the total flow rate of feed sewage and dilution water entering the mixing / aeration tank, \( Q(t) \)
   (l) You can measure the concentration in the mixing / aeration tank and the concentration in the pond
   (m) You can measure the product concentration before it flows into the river
   (n) The feed sewage concentration is \( C_s(t) \), the concentration entering the tank is \( C_1(t) \), the tank concentration is \( C_2(t) \), the pond is \( C_3(t) \), and the final product stream is \( C_5(t) \)

2. Draw a simple feedback control scheme to maintain the flow rate into the mixing / aeration tank.

3. Draw a cascade system to control the final concentration entering the river by adjusting the mixing rate in the aeration tank.

4. Draw a feedforward control scheme assuming the feed concentration varies.

5. Develop a dynamic model of the sewage system assuming the following information:
   
   (a) The volume of the mixing tank \( V_1 \) is 10,000 L and the volume of the pond \( V_2 \) is 100,000 L
   (b) The flow from the mixing tank to the pond is 2,000 L/hr
   (c) The flow from the pond into the mixing tank is 1,000 L/hr
   (d) The tank and pond are constant volume and well-mixed
   (e) The nominal feed flow of entering sewage is 100 L/hr with a concentration is \( C_s(t) \)
   (f) The mixing tank concentration is \( C_1(t) \) and the pond concentration is \( C_2(t) \)
   (g) The concentration entering the river is \( C_5(t) \)
   (h) The volumetric reduction rate in the mixing tank is given as:

   \[ r_1(t) = \frac{1}{10} C_1(t) m(t) \]

   where \( m(t) \) is the normalized mixing rate, \( 0 \leq m(t) \leq 1 \)
   (i) The volumetric reduction rate in the pond is given as:

   \[ r_2(t) = \frac{2}{25} C_2(t) (F(t))^2 \]

   where \( F(t) \) is the normalized chlorine flow rate, \( 0 \leq F(t) \leq 1 \)
   (j) It takes 10 hours for the flow to leave the pond and reach the river
   (k) The feed flow (dilution + sewage) is constant, \( Q(t) = Q_{\text{H2O}} + Q_{\text{Sewage}} = Q \)

6. Determine the steady state tank and pond concentrations, assuming the following:
   
   (a) The feed concentration \( C_s(t) \) is 40 g/L
   (b) The nominal value for \( m(t) \) and \( F(t) \) are both 0.5
7. Linearize your dynamic model at the nominal steady state values. Identify your deviation variables.

8. Put your linear dynamic model in state space form assuming the following:
   (a) Only $C_1(t)$ and $C_2(t)$ are measured as $y_1(t)$ and $y_2(t)$
   (b) The mixing rate deviation is $u_1(t)$
   (c) The chlorine flow rate deviation is $u_2(t)$
   (d) The feed concentration deviation is $u_3(t) = d(t)$

9. What are the eigenvalues of your state space system? Should your system be open-loop stable? **HINT:** $\lambda = -0.259, -0.039$

10. Take the Laplace transform of your model. Develop SISO transfer function models relating the three inputs to $y_1(t)$ and $y_3(t)$. **HINT:** Make sure your poles match your eigenvalues from above.

11. Determine the poles, gains, and zeros of your transfer functions. Are the models stable? Is there inverse response? Do the gains make sense? Is there underdamped response?

12. For a unit increase in the feed concentration, determine the steady state effect on the concentration entering the river using your linear model and the Final Value Theorem.

13. For a unit increase in the initial sewage concentration, determine the analytical response for the concentration entering the river. For simplicity, approximate this transfer function as
   \[ y_3(s) = \frac{1}{s} e^{-10s} \frac{u_3(s)}{(4s + 1)(32s + 1)} \]

14. For an instantaneous increase in the chlorine flow rate ($u_2(t) = \delta(t)$), determine the analytical response for the concentration entering the river. For simplicity, approximate this transfer function as
   \[ y_3(s) = -\frac{5}{2}(5s + 1) e^{-10s} \frac{u_2(s)}{(4s + 1)(32s + 1)} \]

15. For a .1 increase in the mixing rate $u_1$, determine the analytical response for the concentration entering the river $y_3$. Plot this response and determine approximate values for a First-Order-Time-Delay model. For
   \[ y_3(s) = \frac{-1}{2} e^{-10s} \frac{u_1(s)}{(4s + 1)(32s + 1)} \]

16. Determine controller tuning parameters for a PI controller regulating $y_3$ using input $u_1$ using Cohen-Coon tuning parameters.

17. Make a Bode plot between $u_1(t)$ and $y_2(t)$ (Find $AR(\omega)$ and $\phi(\omega)$, sketch the plot) using the approximate model
   \[ y_2(s) = \frac{-1}{(4s + 1)(32s + 1)} u_1(s) \]

18. Make a Bode plot between $u_1(t)$ and $y_3(t)$ (Find $AR(\omega)$ and $\phi(\omega)$, sketch the plot) using the approximate model from problem 15.

19. Find the ultimate gain for a P controller regulating $y_3$ using input $u_1$ using the approximate model.

20. Develop a feedforward controller. Sketch the feedforward block diagram.

21. Develop an IMC controller and sketch the block diagram both in traditional IMC formulation and traditional feedback configuration.

22. Draw the cascade block diagram for $u_1 \rightarrow y_1 \rightarrow y_3$ and determine the CLTF for the system.

23. Draw the 2x2 block diagram for the control system using only $y_1$ and $y_2$.

24. Determine the steady state gain matrix for the system.

25. Using your steady state gain matrix, determine the change in the inputs for a desired decrease in the outputs $y_1$ and $y_2$ of $[-.1, -.1]$

26. Using your steady state gain matrix, determine the RGA and suggest an input pairing. Does this make sense?

27. Develop a decoupling control system for the 2x2 system and draw the block diagram.