A Primer for Communicating Mathematics via Plain Text

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August 26, 2010

Read this if you want to communicate mathematics to me (or anyone) using plain ASCII text. For example, you may want to submit your quiz/exercise/exam answers to me via text-based email or a plain text document, and your answer may include one or more mathematical expressions. There are informal conventions for communicating mathematics via normal (US ASCII) text. These conventions are used throughout the research community, and are loosely based on the LaTeX typesetting system with some programming language constructs thrown in. This brief primer will explain these conventions, allowing you to communicate math via plain text quickly and easily.

These conventions have evolved informally over the years and obey no written hard-and-fast rules. They are meant for human-to-human communication. Some sloppyness/creativity is tolerated, and even encouraged, if the existing conventions are inadequate.

Easy Constructs Easiest are expressions involving Roman letters, the arithmetic operations of addition, subtraction, multiplication, and division, as well as equals, less than, greater than, and parentheses. These expressions can be written practally verbatim. For example, "$2x+6$" can be written as `2x+6` or `2x + 6` or `2 x + 6`. Multiplication is usually just juxtaposition (with optional whitespace in between); you could write `2*x+6`, but you needn't. For another example, "$z < x(2y-4)/w$" is also writable verbatim as `z < x(2y-4)/w`.

Here are some other expressions that can be written entirely verbatim:

- $|x|$ (absolute value of x)
- $(n-2)!$ ($n-2$ factorial)
- $\{x > 0 \mid x \text{ is not an integer}\}$ (the set of all (real) positive x that are not integers)
- $x' := x+1$ (let x' equal $x+1$)
- $x, y, z > 0$ (x, y, and z are all positive)

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- $A[i,j]$ (the i,jth entry of the (two-dimensional) array A)
- $\langle x, y \rangle$ (the pair x, y; write the angle brackets using `<` and `>`, thus getting `<x,y>`)
- $\log x < (\sin y)\exp(iz)$ (transcendental functions can usually be written verbatim, but extra parentheses may be needed to clarify the grouping)

The nonstrict inequalities $\leq$ and $\geq$ can be written as `<=` and `>=`, respectively, but `le` or `leq` are also possible for $\leq$, and `ge` or `geq` for $\geq$. Thus, $x \geq 0$ can be written as either `x >= 0` or `x ge 0`.

The set membership relation $\in$ is usually written as `in`. Thus, $x \in Y$ becomes `x in Y`.

Here is a table of standard mathematical symbols and possibilities for their ASCII text equivalents. Note that many symbols (set relations, for example) can be written in more than one way. I prefer the first of each alternative rendition.

Symbol	Possible renditions
$\leq$	`<=`, `le`, `leq`
$\geq$	`>=`, `ge`, `geq`
$\neq$	`!=`, `ne`, `neq`, `=/=`
$\cong$	`==`, `congto`, `cong` (congruence)
$\equiv$	`equiv`
$\in$	`in`
$\notin$	`notin`
$\subseteq$	`subsetof`, `subset`, `subseteq`
$\not\subseteq$	`notsubsetof`, `not subset of`, etc.
$\supseteq$	`supersetof`, `superset`, `supseteq`
$\cup$	`union`, `cup`
$\cap$	`intersect`, `intersection`, `cap`
$\emptyset$	`emptyset`, `nullset`
$\vee$	`or`, `vee`
$\wedge$	`and`, `wedge`
$\neg$	`not`, `neg`
$\to$	`->`
$\longrightarrow$	`-->`
$\mapsto$	`mapsto`, `|-->`
$\Longrightarrow$	`==>`, `implies`
$\Longleftrightarrow$	`<==>`, `iff`
$\forall$	`forall`, `for each`, etc.
$\exists$	`exists`, `there is`, etc.
∞	`infinity`, `infty`
$\lfloor x \rfloor$	`floor(x)` (the largest integer $\leq x$)
$\lceil x \rceil$	`ceiling(x)` (the smallest integer $\geq x$)
$\sqrt{x}$	`sqrt(x)`
$\vec{x}$	`x-vector`, `vector{x}`, `vec{x}`

Standard Number Systems Number systems with standard notation include $\mathbb{N}$ (natural numbers), $\mathbb{Z}$ (integers), $\mathbb{Q}$ (rational numbers), $\mathbb{R}$ (real numbers), $\mathbb{C}$ (complex numbers), and $\mathbb{H}$ (quaternions). You may use these letters verbatim, provided it is clear what you mean. For example if you use `Z` for the integers, don't use the letter `Z` to mean anything else in the message. To be sure to avoid confusion, you can just write out a set long-hand, i.e., `naturals`, `integers`, `rationals`, `reals`, `complexes`, `quaternions`.

For example, $\{n \in \mathbb{N} : n > 2 \text{ and } n \text{ is prime}\} \subseteq \{n \in \mathbb{N} : n \text{ is odd}\}$ may be written as

```
{ n in N : n>2 and n is prime }  subsetof  { n in N : n is odd }
```

Greek Letters Write Greek letters out long-hand. Thus α is written `alpha`, etc. Lower case Greek letters are written all lower case. Upper case Greek letters, such as Γ, are written with initial capitals (e.g., `Gamma`).

For example, $\sin\theta = 2\Gamma\cos(\alpha + \pi)$ can be written as

```
sin theta = 2 Gamma cos(alpha + pi)
```

Superscripts and Subscripts Use the `^` character (uppercase 6) to signify that the next thing is a superscript. Thus, `x^2` means x^2. If there is more than one token in the superscript (such as 2^{n+1}), then you should surround the entire superscript with some kind of delimiter. Curly brackets (braces) are often used. Thus, 2^{n+1} is written as `2^{n+1}` instead of `2^n+1`; the latter means $2^n + 1$. You may also use brackets or parentheses, e.g., `2^(n+1)`.

Subscripts are similar, but use the `_` (underscore) character instead. Thus x_2 is written as `x_2`. The delimiter requirements are as with superscripts; thus $A_{i,j}$ is written as `A_{i,j}` or `A_(i,j)`.

You may combine both superscripts and subscripts after the same object, but it may help clarity to insert extra parentheses. Thus, x_i^2 could be written as `x_i^2`, but it is clearer to say `(x_i)^2` instead.

Some people don't bother with the underscore for simple subscripts, saying `x2` to mean x_2, for example. This is a ghastly practice, in my opinion.

Sums, Products, Integrals, Etc. To render a sum like

$$\sum_{i=0}^{n-1} 2^i = 2^n - 1,$$

write

```
sum_{i=0}^{n-1} 2^i = 2^n - 1.
```

For products, use `prod` or `product` instead of `sum`. Note that what appears below the summation sign is written as if it were a subscript, and what appears above is like a superscript. Don't say `Sigma` for a sum; "`sum`" is clearer. You can do the same with other sum-like constructs. For definite integrals, use `integral` (you could use `int`, but it may cause confusion); for parameterized unions, use `union` or `UNION`, etc.

Some more examples:

- Write
$$\int_{-\infty}^{\infty} e^{-x^2/2} dx$$
as `integral_{-infty}^infty e^{-x^2/2} dx`.

- Write
$$\sum_{S\subseteq[n]:|S|=k} \prod_{i\in S} i$$
as `sum_{S subsetof [n] : |S|=k} prod_{i in S} i`.

- Write
$$\lim_{n\to\infty} (1 + x/n)^n = e^x$$
as `lim_{n->infty} (1 + x/n)^n = e^x`. You may use “`limit`” instead of “`lim`.”

- Write
$$(\forall a \le b)(\exists c) \left[a \le c \le b \ \wedge \ \int_a^b f(x)dx = f(c)(b-a) \right]$$
as `(forall a<=b)(exists c)[a<=c<=b and integral_a^b f(x)dx = f(c)(b-a)]`.

Binomial Coefficients The binomial coefficient
$$\binom{n}{k} := \frac{n(n-1)\cdots(n-k+1)}{k!} \tag{1}$$
is the number of different ways of choosing k things from a set of n things without regard to order. There are several other ways of expressing this value mathematically: $C(n,k)$, $B(n,k)$, ${}_nC_k$, ${}_nB_k$, etc. I prefer the notation in Equation 1, which is commonly used in the United States, and is pronounced, “n choose k.” In text, render this as `(n choose k)` or `{n choose k}` or some such. The “`choose`” operator has pretty low precedence (even lower that plus or minus), so if you say, e.g.,

```
(n + k choose n - k),
```

I'll interpret it as $\binom{n+k}{n-k}$ rather than, say, $n + \binom{k}{n} - k$.

Fractions Fractions can be a bit problematic. You could render a complicated fraction like
$$\frac{e^{n/2}(a^2+b^2-c^2)^2+1}{\sqrt{n}\sin^2\gamma}$$
using “ASCII art”:

```
e^{n/2} (a^2 + b^2 - c^2)^2 + 1
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      sqrt(n) sin^2 gamma
```

The problem with this is that it may show up crooked if your recipient's email reader uses a font that is pitched differently from yours. This is especially true with a series of several fractions in the same expression—the crookedness compounds so that the fractions on the right of the expression may be unrecognizable.

Although it can look run-on, the lesser of two evils is just to put the numerator and denominator on the same line, separated by the division operator (the forward slash /). Surround the numerator and denominator each with delimiters if there's any doubt about what constitutes the fraction. The fraction above could then be written,

```
[e^{n/2}(a^2 + b^2 - c^2)^2 + 1] / [sqrt(n) sin^2 gamma].
```

Exercises Do these exercises for your own practice. I don't require you to submit them.

Render the following mathematical expressions into ASCII text:

1. $f'(x) = \log x$
2. $\log^* n$
3. $e^{i\pi} = -1$
4. $f : A \to B$
5. $(x_1, \ldots, x_n)$
6. $x \leq y \implies f(x) \leq f(y)$
7. $[P(0) \wedge (\forall n)(P(n) \to P(n+1))] \to (\forall n)P(n)$
8. $\varphi := (x_1 \vee \neg x_2) \wedge (\neg x_3 \vee x_4)$
9. $\forall \varepsilon > 0,\ n^\varepsilon = \Omega(\lg n)$
10. $$\{X_i \mid i \in I\} \subseteq T \implies \bigcup_{i \in I} X_i \in T$$
11. $$\sum_{i=1}^{n} \sum_{j=1}^{i} j^2$$
12. $$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}$$
13. $$\sum_{i=1}^{n} i^k = \Theta(n^{k+1})$$

14.

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

15.

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

(the symbol ζ is the Greek letter zeta)