From now on, homework will be due anytime on or before the due date. In other words, you do not need to hand it to me in class, although you can still do that if you wish. If I’m not in my office, you can just slip it under the door.

Reading

- KLM, Sections 3.2 and 3.4. We will need the Section 3.5 material at some point later, so it won’t hurt to read through that as well.
- KLM, Sections 4.1 to 4.3
- Course notes as needed.

Written Exercises

1. I asserted in class that for all \( y = (y_1, \ldots, y_n) \in \{0, 1\}^n \),

\[
H^{\oplus n}|y\rangle = 2^{-n/2} \sum_{z \in \{0,1\}^n} (-1)^{y \cdot z} |z\rangle,
\]

where \( H \) is the 1-qubit Hadamard operator and \( y \cdot z \) is the standard dot product of vectors. Prove this fact by induction on \( n \geq 1 \).

2. KLM Exercises 3.3.1, 3.4.2, 4.2.1. Do 4.2.1 before doing the next exercise.

3. KLM Exercise 4.2.2. [Hint: Doing this with spherical coordinates will be messy, because expressing any rotation not around the z-axis is complicated using them. Instead, use the density operator approach, which works with the cartesian coordinates. That is, consider an arbitrary state

\[
\rho = \frac{1}{2}(I + \alpha_x X + \alpha_y Y + \alpha_z Z),
\]

where \( X, Y, Z \) are the Pauli spin matrices, and \((\alpha_x, \alpha_y, \alpha_z) \in \mathbb{R}^3 \). Applying, say, the unitary operator \( R_x(\theta) \) to \( \rho \) results in a state

\[
\rho' = R_x(\theta)\rho R_x(\theta)^* = \frac{1}{2}(I + \alpha'_x X + \alpha'_y Y + \alpha'_z Z)
\]

Now show that the column vector \((\alpha'_x, \alpha'_y, \alpha'_z)\) can be obtained by multiplying the column vector \((\alpha_x, \alpha_y, \alpha_z)\) on the left by a certain \(3 \times 3\) real matrix. Do the same for \( R_y(\theta) \). For all of this, use Exercise 4.2.1 and the multiplication relationships among the Pauli matrices. By the way, note that \( R_x(\theta)^* = R_x(-\theta) \), and similarly for \( R_y(\theta)^* \) and \( R_z(\theta)^* \).]