CSCE 790, Spring 2020, Homework 1 (due 2/5/2020)

Reading

KLM stands for Kaye, Laflamme, & Mosca.

- KLM, Chapter 1
- KLM, Chapter 2, Sections 2.1–2.2
- Course notes, first three lectures
- Recall how we defined the inner product over $\mathbb{C}^n$: For vectors $u = (u_1, \ldots, u_n)^T$ and $v = (v_1, \ldots, v_n)^T$ in $\mathbb{C}^n$,\(^1\) the inner product is defined as

$$\langle u, v \rangle := \sum_{k=1}^{n} u_k^* v_k \in \mathbb{C}.$$

Exercises

- Course notes, Exercises 2.1, 2.4, 2.5, 2.6, 2.8
- Verify the defining properties of the inner product over $\mathbb{C}^n$: for all $u, v, w \in \mathbb{C}^n$ and $\alpha \in \mathbb{C}$,

1. $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$ and $\langle u, \alpha v \rangle = \alpha \langle u, v \rangle$. (Linearity in the second argument)
2. $\langle u, v \rangle = \langle v, u \rangle^*$. (Conjugate symmetry)
3. $\langle u, u \rangle \geq 0$ with equality holding if and only if $u = 0$ (i.e., iff $u$ is the zero vector).\(^2\) (Positive definiteness)

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\(^1\)The $T$ superscript means “transpose.” This is because $u$ and $v$ are really column vectors, and displaying them as such in the running text takes up too much room.

\(^2\)If we every compare a complex number $z$ with a real constant, e.g., $z \geq 0$, then this tacitly implies that $z$ itself is real.
• Show that any orthogonal set of nonzero vectors is linearly independent. [Hint: Let $v$ be any linear combination of such vectors, and consider $\langle v, v \rangle$. You’ll need the fact that $\langle \cdot, \cdot \rangle$ is positive definite.]

• Extra credit: KLM Exercise 2.3.1, Course notes, Exercises 2.10, 3.3

Note: The notations used in the book and in my course notes are pretty similar, but beware of the following two discrepancies:

1. I denote the adjoint of an operator $A$ by $A^*$, which is the general convention in pure math. The KLM book uses $A^\dagger$ to denote the adjoint of $A$, which is used more in the physics community. (Discrepancies like this are inevitable in such a multidisciplinary subject.)

2. I don’t introduce Dirac notation until later, whereas KLM uses it early on.