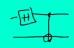
Quantum Teleportation

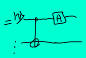
Alice $\longleftarrow |\Phi^+\rangle \longrightarrow$ Bob

$|\psi\rangle$ arbitrary 1-qubit state $\quad = \cancel{\frac{|00\rangle + |11\rangle}{\sqrt{2}}} \quad |\psi\rangle$

$|\Phi^+\rangle$ called an EPR pair

Let $B :=$ 

Alice: Has $|\psi\rangle$ on first qubit & $\frac{1}{2}$ the EPR pair on her 2nd qubit.

Alice (1) applies $B^* =$ 

to her 2 qubits

(2) Measures each qubit in the computational basis, gets 2 bits b_1, b_2

sends b_1, b_2 to Bob

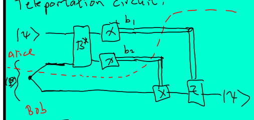
Bob:

if $b_2 = 1$, applies X to his qubit.

if $b_1 = 1$, applies Z to his qubit.

Now Bob's qubit is in state $|\psi\rangle$.

Teleportation circuit:



Analysis

Let $|\psi\rangle := \alpha|0\rangle + \beta|1\rangle$

some $\alpha, \beta \in \mathbb{C}$ s.t. $|\alpha|^2 + |\beta|^2 = 1$.

Can check that

$$|00\rangle = (|\Phi^+\rangle + |\Phi^-\rangle)/\sqrt{2}$$
$$|01\rangle = (|\Psi^+\rangle + |\Psi^-\rangle)/\sqrt{2}$$
$$|10\rangle = (|\Psi^+\rangle - |\Psi^-\rangle)/\sqrt{2}$$
$$|11\rangle = (|\Phi^+\rangle - |\Phi^-\rangle)/\sqrt{2}$$

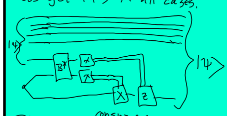
Initial state is

$$|\psi\rangle \otimes |\Phi^+\rangle = \frac{1}{\sqrt{2}}(\alpha|0\rangle + \beta|1\rangle) \otimes (|00\rangle + |11\rangle)$$
$$= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)$$
$$= \frac{1}{2}\Big(\alpha(|\Phi^+\rangle + |\Phi^-\rangle)|0\rangle + \alpha(|\Psi^+\rangle + |\Psi^-\rangle)|1\rangle + \beta(|\Psi^+\rangle - |\Psi^-\rangle)|0\rangle + \beta(|\Phi^+\rangle - |\Phi^-\rangle)|1\rangle\Big)$$
$$= \frac{1}{2}\Big(|\Phi^+\rangle(\alpha|0\rangle + \beta|1\rangle) + |\Psi^+\rangle(\alpha|1\rangle + \beta|0\rangle) + |\Psi^-\rangle(\alpha|1\rangle - \beta|0\rangle) + |\Phi^-\rangle(\alpha|0\rangle - \beta|1\rangle)\Big)$$
$$\xmapsto[\text{Alice}]{B^*} \frac{1}{2}\Big(|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle)\Big)$$

Alice measures qubits 1 & 2, gets

b_1, b_2:	Bob:
$b_1 b_2 = 00$:	$\alpha\|0\rangle + \beta\|1\rangle \xmapsto{I} \|\psi\rangle$
$b_1 b_2 = 01$	$\alpha\|1\rangle + \beta\|0\rangle \xmapsto{X} \|\psi\rangle$
$b_1 b_2 = 10$	$\alpha\|0\rangle - \beta\|1\rangle \xmapsto{Z} \|\psi\rangle$
$b_1 b_2 = 11$	$\alpha\|1\rangle - \beta\|0\rangle \xmapsto{ZX} \|\psi\rangle$

Bob get $|\psi\rangle$ in all cases.



Teleportation ~~destroys~~ consumes one EPR pair per qubit teleported.

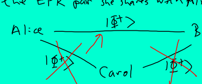
A Bell state is 1 ebit (entangled bit)

Entanglement swapping
(kind of entanglement tranformation)

Alice $|\psi\rangle$ —— $|\Phi^+\rangle$ —— Carol —— $|\Phi^+\rangle$ —— Bob

Carol is a quantum intermediary with Alice & Bob as clients

To share an EPR directly with Bob, Alice asks Carol to teleport to Bob Carol's half of the EPR pair she shares with Alice:



Simon's Problem:

Boolean function $f : \{0,1\}^n \to \{0,1\}^m$ such that $\exists s \in \{0,1\}^n \; \forall x, y \in \{0,1\}^n$
$f(x) = f(y) \iff x \oplus y = s$. **Task**: find s.] promise

s is uniquely determined by f;

$s = 0^n$:

$$f(x) = f(y) \iff x \oplus y = 0^n$$
$$\iff x = y$$

$\iff f$ is one-to-one

$s \neq 0$: $f(x) = f(y) \iff x \oplus y = s$

$$\iff y = x \oplus s$$

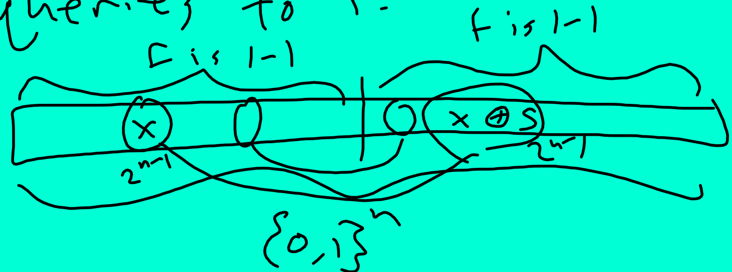
So $(x := 0^n)$: $f(0) = f(y) \iff y = s$

$\therefore$ s is the unique string $\neq 0$ such that $f(s) = f(0)$

f is 2-to-1:

$\forall x,\ f(x) = f(x \oplus s) \neq f(y)$ for any $y \neq x \oplus s$

<u>Classically</u>: Need $2^{n-1} + 1$ queries to f.



Random queries to f:

need $\sqrt{2^n} = 2^{n/2}$ queries to f to find a collision with high probability,

$$f(x) \neq f(y)$$
$$s \neq x \oplus y$$