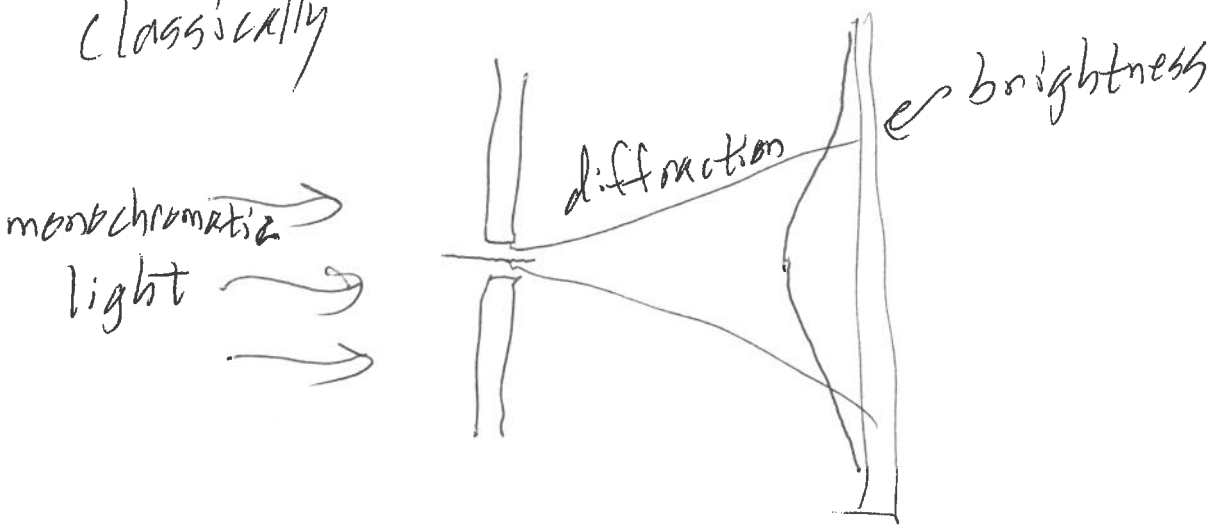
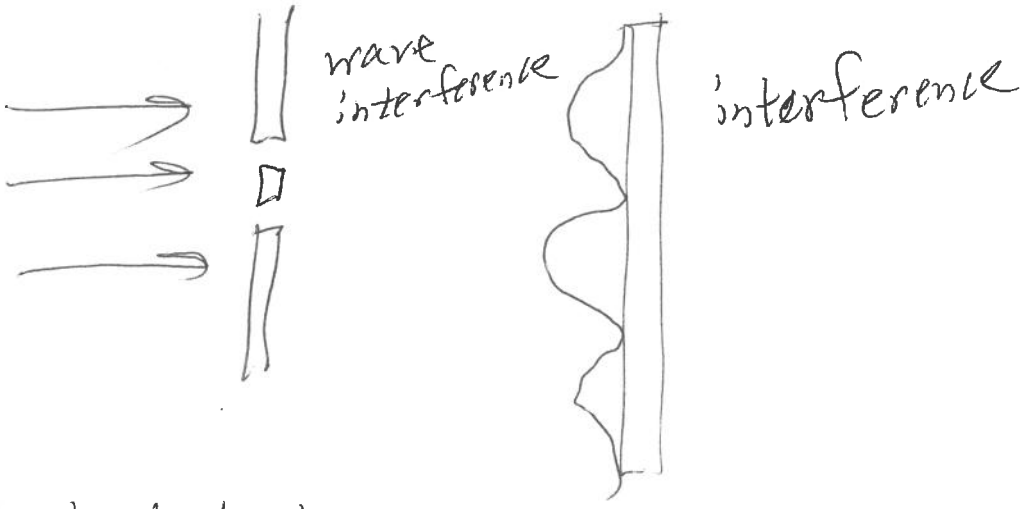
CSCE 785
9/12/2023

Quantum weirdness
What is a qubit?

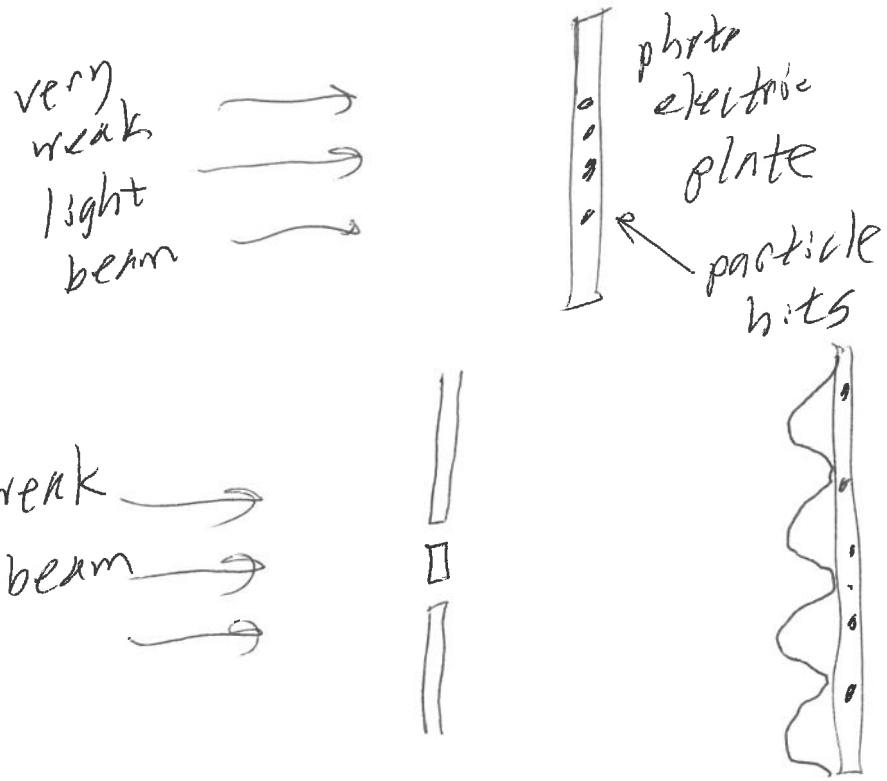
2 experiments: Double slit
Stern-Gerlach

classically





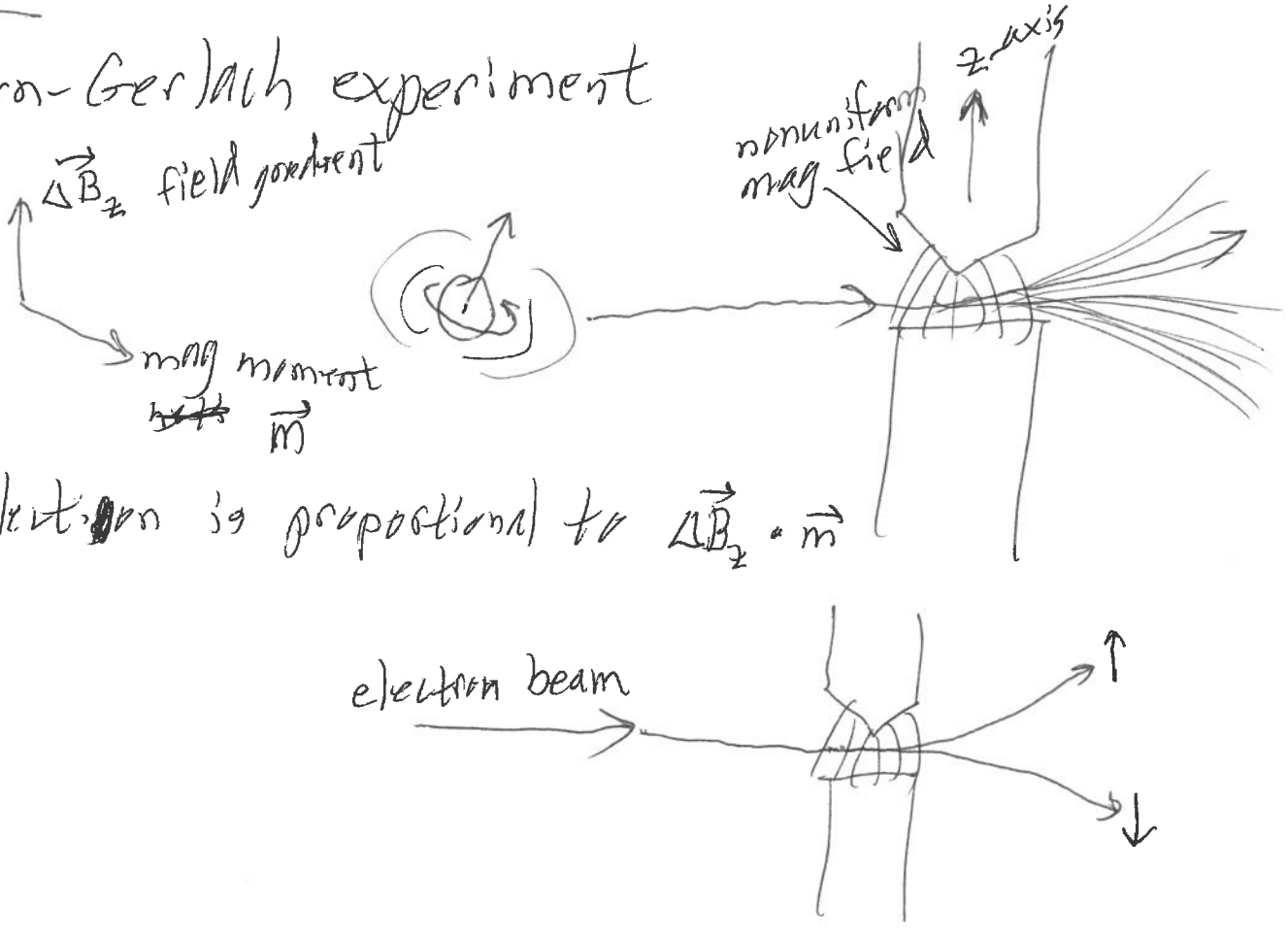
particle trajectories



Nondeterminism/randomness is inherent in nature.

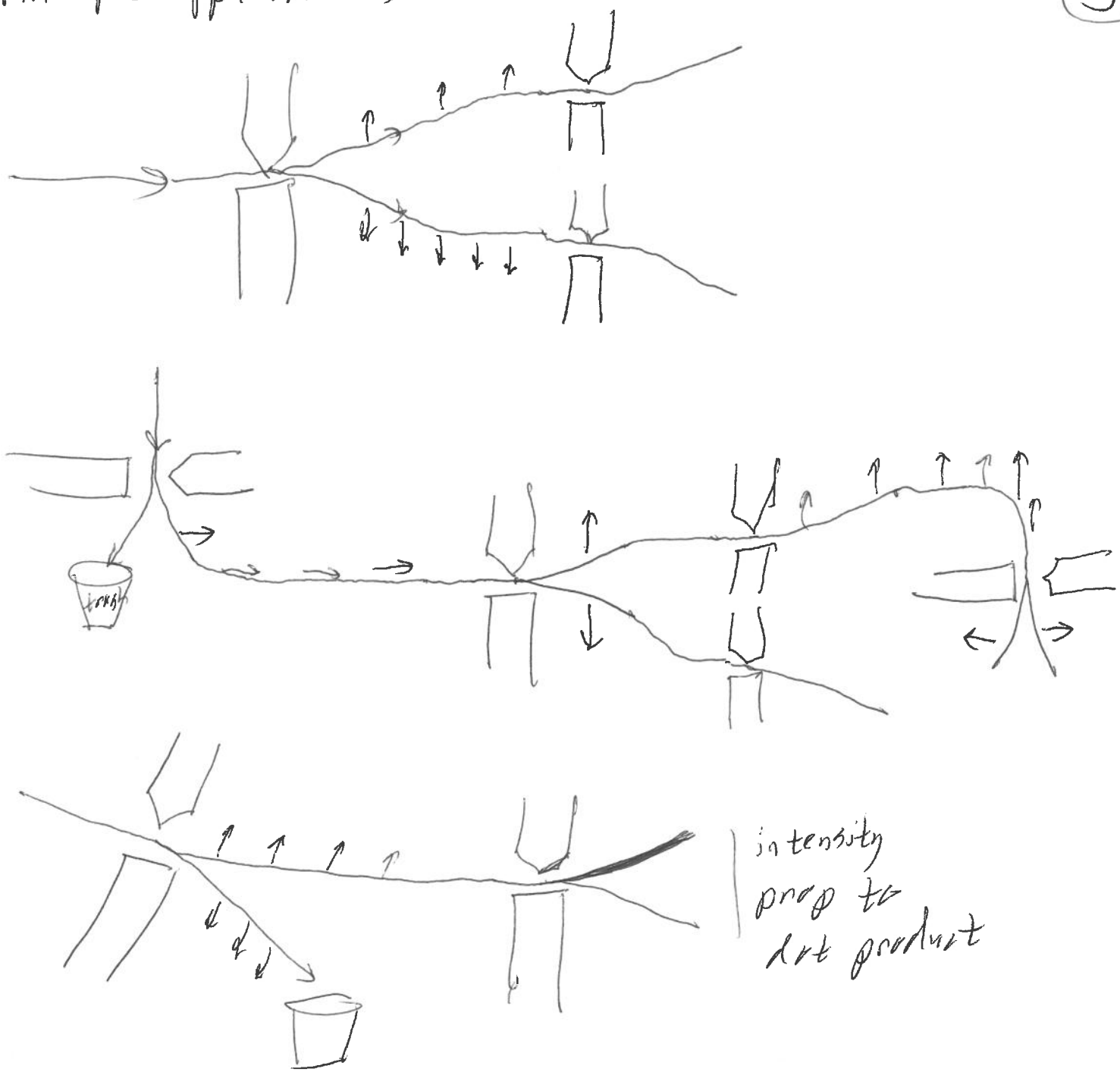
Interference/cancellation of probability distributions

Stern-Gerlach experiment



deflection is proportional to $\nabla \vec{B}_z \cdot \vec{m}$

Multiple apparatuses



QM postulates

A bounded physical system S (e.g., electron spin) is modeled by a $\mathbb{C}$-space $\mathcal{H}$. At any given "time" S is in some state, modeled (for now) by a unit vector $u \in \mathcal{H}$.

Electron spin system: $\mathcal{H}$ is 2-dimensional ("two level system"), and we fix an orthonormal basis $\{|\uparrow\rangle, |\downarrow\rangle\}$ for $\mathcal{H}$

$|\uparrow\rangle$: spin up; $|\downarrow\rangle$: spin down

A general electron spin is a unit vector

$$|\varphi\rangle := \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

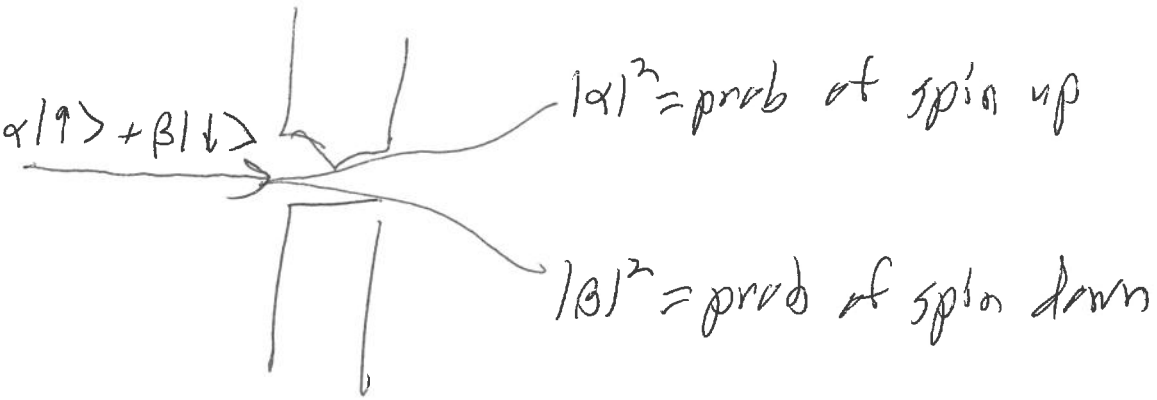
some $\alpha, \beta \in \mathbb{C}$

$|\varphi\rangle$ is a unit vector: $\||\varphi\rangle\| = 1$

$$\||\varphi\rangle\| = \sqrt{|\alpha|^2 + |\beta|^2} = 1$$

$$\therefore |\alpha|^2 + |\beta|^2 = 1$$

$\therefore (|\alpha|^2, |\beta|^2)$ is a probability distribution



$$|\rightarrow\rangle := \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$

$$|\leftarrow\rangle := \frac{1}{\sqrt{2}}|\uparrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\rangle$$

orthogonal

$$|x\rangle := \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{i}{\sqrt{2}}|\downarrow\rangle$$

"spin in" into the paper

$$|\odot\rangle := \frac{1}{\sqrt{2}}|\uparrow\rangle - \frac{i}{\sqrt{2}}|\downarrow\rangle$$

"spin out" out from the paper

orthogonal