CSCE 750 Quiz 1
January 28, 2004

Show that for any real constants $a$ and $b$, where $b > 0$,

$$(n + a)^b = \Theta(n^b).$$

CSCE 750 Quiz 2
February 4, 2004

Show that

$$\sum_{k=1}^{n} \frac{1}{k^2}$$

is bounded above by a constant.

CSCE 750 Quiz 3
February 11, 2004

Argue that the solution to the recurrence

$$T(n) = T(n/3) + T(2n/3) + cn,$$

where $c$ is a constant, is $\Omega(n \lg n)$ by appealing to a recursion tree.

CSCE 750 Quiz 4
February 18, 2004

Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for sufficiently small $n$. Make your bounds as tight as possible, and justify your answers.

1. $T(n) = 10T(n/3) + n^2$,
2. $T(n) = \sqrt{n}T(\sqrt{n}) + n$.

CSCE 750 Quiz 5
February 25, 2004

Show that, with the array representation for storing an $n$-element heap, the leaves are the nodes indexed by $[n/2] + 1, [n/2] + 2, \ldots, n$.

CSCE 750 Quiz 6
March 3, 2004

Consider the following version of Quicksort, which avoids tail recursion:

```cpp
QUICKSORT'(A, p, r)

while p < r do
    q ← PARTITION(A, p, r)
    QUICKSORT'(A, p, q - 1)
    p ← q + 1
```

Modify the code for QUICKSORT' so that the worst-case stack depth is $\Theta(\lg n)$ (where $n = r - p + 1$). Maintain the $O(n \lg n)$ expected running time of the algorithm.
CSCE 750 Quiz 7
March 17, 2004
You are given $n$ red jugs and $n$ blue jugs such that no two jugs of the same color have the same size, but each red jug is the same size as some blue jug. You want to find all $n$ pairs of equal-size jugs.

The only comparison you are allowed is to compare the sizes of any two jugs of different color, to find which jug is bigger or if they are the same size.

Give a randomized algorithm to solve this problem whose expected number of comparisons is $O(n\lg n)$. Pseudocode is not necessary; a high-level English description is fine. You need not prove that your answer is correct. [Hint: use divide-and-conquer recursion.]

CSCE 750 Quiz 8
March 24, 2004
Professor Rosenkrantz flips a fair coin twice. Professor Gildenstern flips a fair coin three times. What is the probability that Professor Rosenkrantz obtains more heads than Professor Gildenstern?

CSCE 750 Quiz 9
March 31, 2004
Consider a red-black tree formed by inserting $n$ nodes with RB-INSERT into an initially empty tree. Argue that if $n > 1$, the tree has at least one red node.

CSCE 750 Quiz 10
April 7, 2004
Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $(10, 5, 12, 3, 6, 50, 5)$.

CSCE 750 Quiz 11
April 14, 2004
A sequence of $n$ operations is performed on a data structure. The $i$th operation costs $i$ if $i$ is an exact power of 2, and 1 otherwise. Use a potential method of analysis to determine the amortized cost per operation.

CSCE 750 Quiz 12
April 21, 2004
The Connected-Components algorithm, reproduced below from the textbook, takes an undirected graph $G = (V, E)$ as input.

Connected-Components($G$)
for each vertex $v \in V$ do
    Make-Set($v$)
for each edge $(u, v) \in E$ do
    if $\text{Find-Set}(u) \neq \text{Find-Set}(v)$ then
        Union($u, v$)

Suppose that $G$ has $k$ connected components. How many times is $\text{Find-Set}$ called during the execution of Connected-Components($G$)? How many times is $\text{Union}$ called? Express your answers exactly (i.e., not just big-$\Theta$) in terms of $|V|$, $|E|$, and $k$. Simplify your answers as much as possible, and include a brief explanation.
CSCE 750 Quiz 13
April 28, 2004
Consider the following decision problem:

MINIMUM SUM OF SQUARES
INSTANCE: Finite set $A$, “size” $s(a) \in \mathbb{Z}^+$ for each $a \in A$, positive integers $K$ and $J$.
QUESTION: Can the elements of $A$ be partitioned into $K$ disjoint sets $A_1, A_2, \ldots, A_K$ such that

$$\sum_{i=1}^{K} \left( \sum_{a \in A_i} s(a) \right)^2 \leq J ?$$

Describe a polynomial reduction from PARTITION to MINIMUM SUM OF SQUARES. [Hint: show that the former is a restriction of the latter.]