# Mergeable Heaps (min heaps)

Basic Ops:

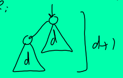
- Insert$(H, x)$
- FindMin$(H)$
- DeleteMin$(H)$
- DecreaseKey$(H, x, k)$
- Delete$(H, x)$
- Merge$(H_1, H_2)$ — combine heaps $H_1$ & $H_2$ into a single heap ($H_1$ & $H_2$ are destroyed)

2 implementations:

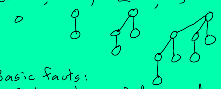
- Binomial Heaps
- Fibonacci Heaps

A binomial tree of degree $d \geq 0$ is defined inductively as follows:

1. A binomial tree of degree 0 is a single node (the root)
2. For $d \geq 0$, a binomial tree of degree $d+1$ is obtained from 2 binomial trees of degree $d$ by making the root of one tree the leftmost child of the root of the other tree.




$d = 0$, 1, 2, 3



Basic facts:

- A binomial tree of degree $d$ has height $d$ & size $2^d$ (by induction on $d$)
- Every node in a binomial tree is the root of a binomial (sub) tree
- The root of a binomial tree of degree has exactly $d$ many children (induction on $d$) (degree of a node = # of its children)
- The children of a node of degree $d$ have degrees $d-1, d-2, \ldots, 0$ from left to right.

All easily provable by induction on $d$.

For $0 \leq i \leq d$, the $i$th level of a binomial tree of degree $d$ has exactly $\binom{d}{i}$ many nodes, where $\binom{d}{i} = \frac{d!}{i!(d-i)!}$ is the binomial coefficient "$d$ choose $i$"

[induction on $i$ using the "Pascal's triangle" recurrence:

$$\binom{d}{0} = \binom{d}{d} = 1 \quad \text{(boundary conditions)}$$

for $0 < i < d$

$$\binom{d}{i} = \binom{d-1}{i-1} + \binom{d-1}{i}$$ ]

A binomial heap is a sequence of binomial trees of strictly increasing degree. data (key, satellite info) are kept in the nodes. Each tree in the sequence has keys in minheap order.

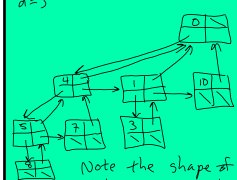
Nodes have the following attributes:

- data — as above
- leftmost_child (left) — pointer to the leftmost child of the node.
- right_sibling (right) — pointer to the nodes immediate right sibling (on the same level)
- parent — points to the parent
- degree — the degree of the node

Ex:

| | |
|---|---|
| data | parent |
| left | right |

$d = 3$




Note the shape of a binomial heap is uniquely determined by its size.