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 Prove: L is not regular.
 Proof: Suppose L is regular. Then L is regular. (closed prop of reg lang)
 Then $L \cap \{0^n\}$ is regular (intersection of reg lang)
 But $L \cap \{0^n\} = \{0^n : n \text{ is even and } n > 0\}$
 $= \{0^{2k} : k \geq 1\}$
 which is not regular by the pumping lemma. \square
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