

CSCE 551/MATH 562, Homework 1

The numbered exercises are from the textbook, written out for the purpose of comparing your book version's exercises with mine. (Note: "state diagram" is the same as "transition diagram.") You should do the exercises as worded below.

Exercise 1.4: Each of the following languages is the intersection of two simpler languages.

In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

- c. $\{w \mid w \text{ has an even number of } a\text{'s and one or two } b\text{'s}\}$
- e. $\{w \mid w \text{ starts with an } a \text{ and has at most one } b\}$

Exercise 1.5: Each of the following languages is the complement of a simpler language.

In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

- d. $\{w \mid w \text{ is any string not in } a^*b^*\}$
- f. $\{w \mid w \text{ is any string not in } a^* \cup b^*\}$

Exercise 1.6: Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0, 1\}$.

- c. $\{w \mid w \text{ contains the substring } 0101 \text{ (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$
- l. $\{w \mid w \text{ contains an even number of } 0\text{'s or contains exactly two } 1\text{'s}\}$

Exercise 1.7: Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{0, 1\}$.

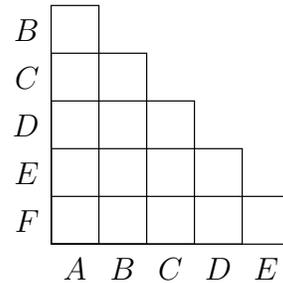
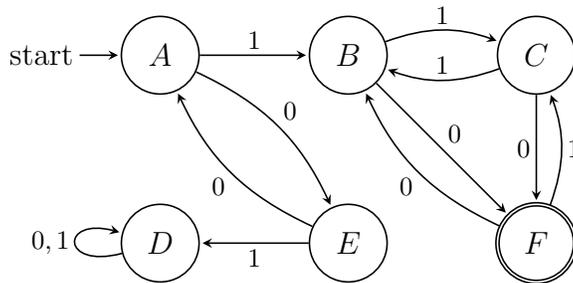
- b. The language of Exercise 1.6c with five states
- c. The language of Exercise 1.6l with six states

Exercise 1.16: Use the construction given in Theorem 1.39 to convert the following two nondeterministic finite automata to equivalent deterministic finite automata.

b. [Given in tabular form:]

	a	b	ε
$\rightarrow 1$	{3}	\emptyset	{2}
*2	{1}	\emptyset	\emptyset
3	{2}	{2, 3}	\emptyset

Not in Textbook 1: Consider the DFA A (below left) over the alphabet $\{0, 1\}$:



1. Fill in the distiguishability table to the right with X in each entry corresponding to a pair of distinguishable states.
2. Draw (as a transition diagram) the minimal DFA equivalent to A .

Not in Textbook 2: Consider the following DFA A (given in tabular form):

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

Show that $L(A)$ is the language of all binary representations of natural numbers that are multiples of 3. (Here we assume ε represents the number zero, which is a multiple of 3.) Hint: Prove the stronger statement that, for $k \in \{0, 1, 2\}$, the computation of A on input string w ends in state q_k iff w represents a number whose remainder is k when divided by 3. Make this argument by induction on $|w|$.