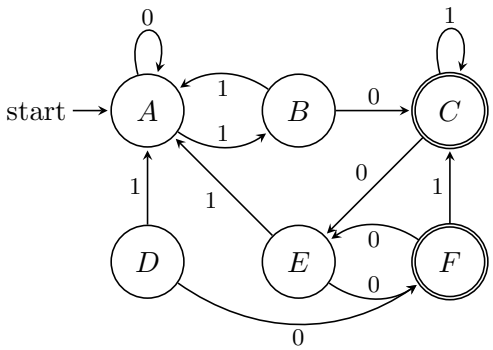
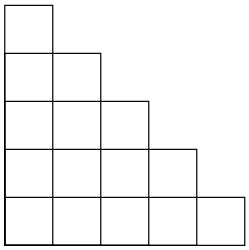
Name:

Sample Final Examination Questions
CSCE 551/MATH 562
Spring Semester 2021

Problem 1 (DFA Minimization — Essential)
Consider the following DFA A over the alphabet $\{0, 1\}$:

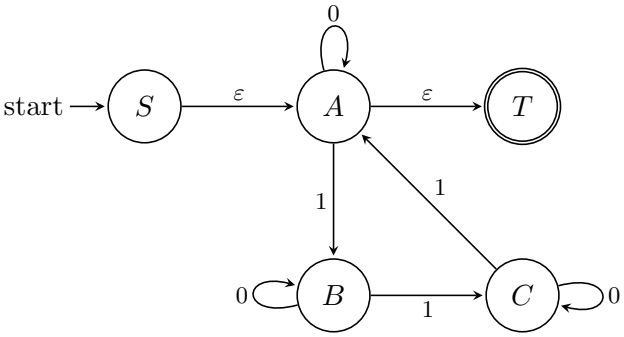


(1) Give the table T of distinguishabilities for A. Show only the proper lower triangle of T, that is, fill in the following table with X in each entry corresponding to a pair of distinguishable



(2) Draw the minimal (4-state) DFA equivalent to A.

Problem 2 (NFA Conversion)
Consider the following clean NFA N:



(1) Using the state-elimination method described in the book or in class, convert N into an equivalent regular expression.

(2) Using the set-of-states method, convert N into an equivalent DFA, only giving those states that are reachable from the start state S. Do not perform any simplifications such as merging indistinguishable states.

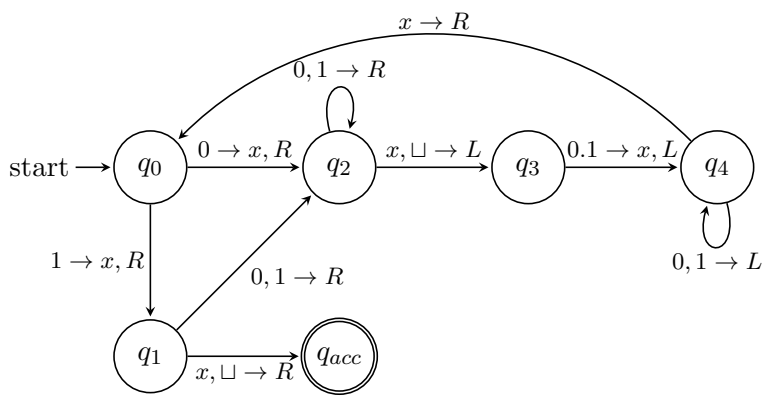
Problem 3 (Closure Properties of Regular Languages — Essential)
The alphabet in this problem contains the symbol 0. For any word w on our alphabet let NoZero(w) result from w by deleting all the 0's in w and then closing up the gaps. For any language L let NoZero(L) $= \{\text{NoZero}(w) \mid w \in L\}$. Prove that if L is a regular language, then so is NoZero(L).

Problem 4 (The Pumping Lemma)
Show that the language

$$L = \{w \in \Sigma^* \mid \text{no prefix of } w \text{ has more 0's than 1's}\}$$

is not pumpable (hence not regular). [Note: a string w is always a prefix of itself.]

Problem 5 (Turing Machines — Essential)
Consider a standard, one-tape Turing machine M with input alphabet $\{0, 1\}$, tape alphabet $\{0, 1, x, \sqcup\}$ where $\sqcup$ is the blank symbol, and the following transition diagram:



Give the sequence of IDs (configurations) of the complete computation of M on each of the three inputs 011, 10, and 0. (Hint: M decides the language of all binary strings of odd length whose middle symbol is 1, so the first input is accepted while the other two are rejected.)

Problem 6 (Decidability and Computability)
Fix some enumerator E that enumerates an infinite language. Let A be the language of all strings w so that

- E prints w at least once, and
- when w is first printed, it is longer than any string E has printed before.

Show that A is infinite and decidable. For the latter, give an explicit decision procedure for A.

Problem 7 (Undecidability and Noncomputability — Essential)
Let f be a function that has the following property: for any TM M and string w such that M accepts w, $f(\langle M, w\rangle)$ outputs a number t such that M accepts w in less than t steps. (If M does not accept w, then $f(\langle M, w\rangle)$ could be any natural number.)

Show that no such f can be computable. [Hint: Show that if f were computable, then one can decide $\mathrm{A_{TM}}$.]

Problem 8 (Enumerability/Turing Recognizability)
Let f be a computable function. Show that range(f) is Turing-recognizable. (Here, range(f) is defined as $\{y \mid (\exists x)\ f(x) = y\}$).

Problem 9 (Reducibility — Essential)
Let $B := \{\langle M \rangle \mid M \text{ is a TM and } 010 \in L(M) \text{ and } 011 \notin L(M)\ \}$. Construct a mapping reduction from $\overline{\mathrm{A}_{\mathrm{TM}}}$ to B.

Problem 10 (Polynomial-Time Computability)
Let

$$L := \{\langle M, w \rangle \mid M \text{is a DFA that accepts } w^n \text{ for all natural numbers } n\}.$$

Show that $L \in \mathbf{P}$ by giving a polynomial time decision procedure for L.

Problem 11 (NP and NP-Completeness — Essential)
The language

$$\mathrm{VC} := \{\langle G, k \rangle \mid G \text{ is a graph that has a vertex cover of size } k\}.$$

VC is clearly in **NP**. Show that VC is **NP**-complete by giving a polynomial time reduction from CLIQUE.

Problem 12 (Space-Bounded Computation)
Prove that if TBFQ $\in$ **NP**, then **NP** = **PSPACE**.