Context-Free Grammar
(CFG or grammar)

- terminals (abstract or lexical)
- nonterminals (abstract)
- productions (rules)

Grammar for balanced parentheses:

\[ S \rightarrow (S)S \]
\[ S \rightarrow ()S \]
\[ S \rightarrow (S) \]

where \( S \) represents any string of balanced parentheses.

A derivation from some nonterminal \( A \) is a sequence of the form

\[ A \rightarrow a_1 \rightarrow a_2 \rightarrow \ldots \rightarrow a_n \]

where each \( a_i \) is a string of nonterminals and terminals ("sentential form") and each \( a_{i+1} \) results from \( a_i \) by replacing some occurrences of a nonterminal \( V \) with the right-hand side of a production whose left-hand side is \( V \).

Production (in general):

\[ V \rightarrow X_1X_2X_3 \ldots X_n \]

where \( V \) is a nonterminal and \( X_1, \ldots, X_n \) are nonterminals or terminals (\( n \geq 0 \)).

A context-free grammar is a list of productions:

\[ S \rightarrow \epsilon \]
\[ S \rightarrow (S)S \]
\[ S \rightarrow (S) \]
\[ S \rightarrow ()S \]
\[ S \rightarrow (S) \]

A starting symbol is interpreted as the start symbol.

(Convention: the start symbol will be the last in the list of productions in the text.)

A derivation is complete if (it's finite, and)

- it starts with the start symbol
- and ends with a string consisting only of terminals.

Let \( G \) be a grammar:

\[ N = \text{set of nonterminals} \]
\[ T = \text{set of terminals} \]
\[ \Delta = \text{set of productions} \]

\[ |N| + |T| = \beta \]

Let \( S \) be the start symbol.

A string \( \alpha \) is terminal if \( \alpha \) consists only of terminals.

A string \( \alpha \) is generated by \( G \) (Chomsky from a complete derivation ending in \( \alpha \).)
term (T) is a string of one or more factors (F) separated by \( \times, \div \), \% (grouped from left to right)

\[
T \rightarrow F \mid T \times F \mid T \div F \mid T \% F
\]

\[
F \rightarrow c \mid v \mid (E)
\]

This is the standard unambiguous grammar for arithmetic expressions.