CSCE 355
1/29/2024

DFA minimization

Given a DFA $A = \langle Q, \Sigma, \delta, q_0, F \rangle$, goal is to find a DFA A_{min} such that $L(A_{min}) = L(A)$ and A_{min} has the fewest possible states. (A_{min} is unique given A).

2 steps:

1) Remove from A all states not reachable from q_0. (A DFA where all states are reachable we'll call sane.)
   Such a DFA is equivalent to A.

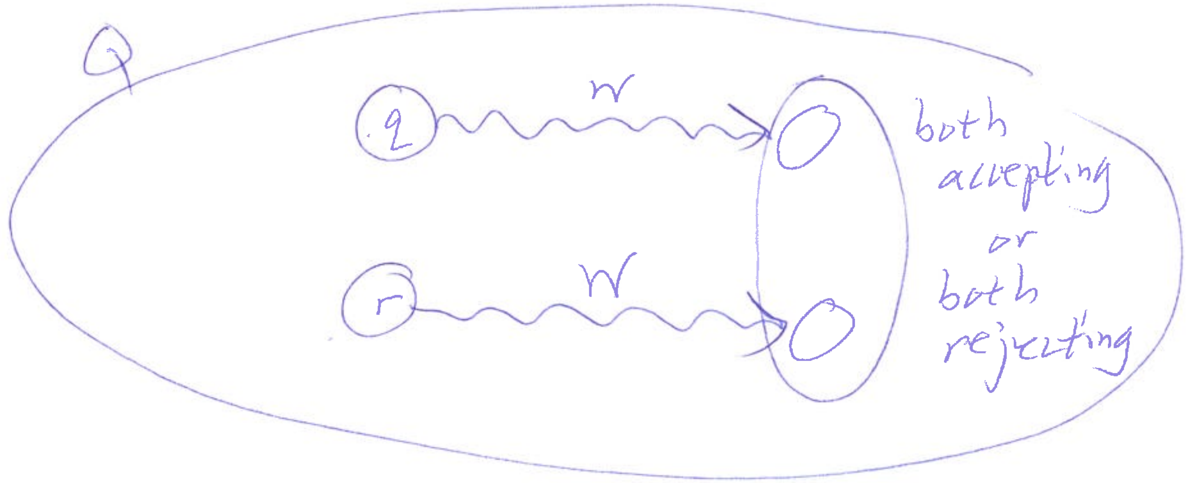
2) Merge clusters of indistinguishable states into single state.

Definition: Let A (sane) be as above and let $q \in Q$ be a state of A. Define the DFA

$$A_q := \langle Q, \Sigma, \delta, \underbrace{q}_{\text{start state}}, F \rangle$$

Note. $A_{q_0} = A$.

<u>Def</u>: $q, r \in Q$ are <u>indistinguishable</u>
if $L(A_q) = L(A_r)$.

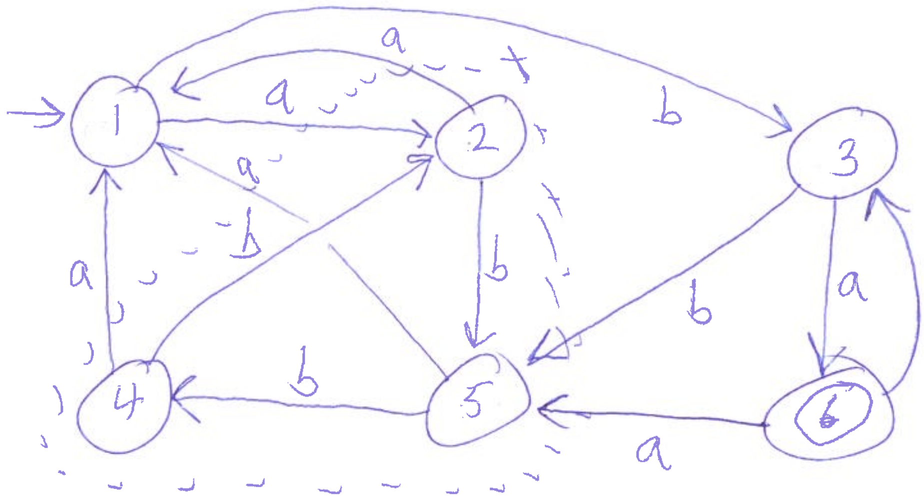


So q, r are <u>distinguishable</u> if $L(A_q) \neq L(A_r)$,
that is, there exists a string $w \in \Sigma^*$
such that, reading w from q gives a different
result from reading w from r. (One accepting,
the other rejecting.) $\boxed{w \text{ distinguishes } q \text{ from } r}$

To perform Step 2, find pairs of distinguishable
states, until we can't find any more. Pairs left over
are guaranteed indistinguishable.

Ex:

$\Sigma = \{a, b\}$



Rule 1: Every accepting state is distinguishable from every rejecting state. (distinguished by $w = \varepsilon$)

Mark X for each dist. pair found

	1	2	3	4	5
2	X				
3	X	X			
4	X		X		
5	X		X		
6	X	X	X	X	X

Rule 2: q & r are distinguishable if there exists $s \in \Sigma$ such that

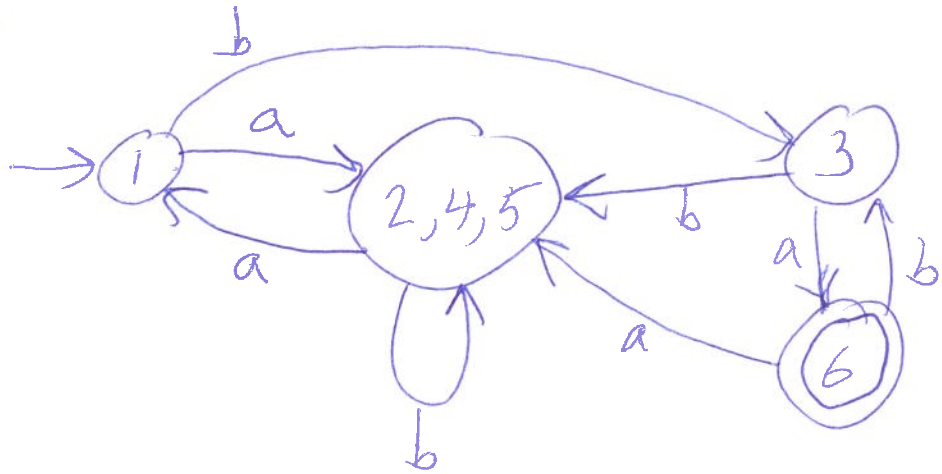
$\delta(q, s)$ & $\delta(r, s)$ are distinguishable.

Fact: Rules 1 & 2 suffice for finding <u>all</u> distinguishable pairs.

if (t, u) are dist. then (q, r) are dist

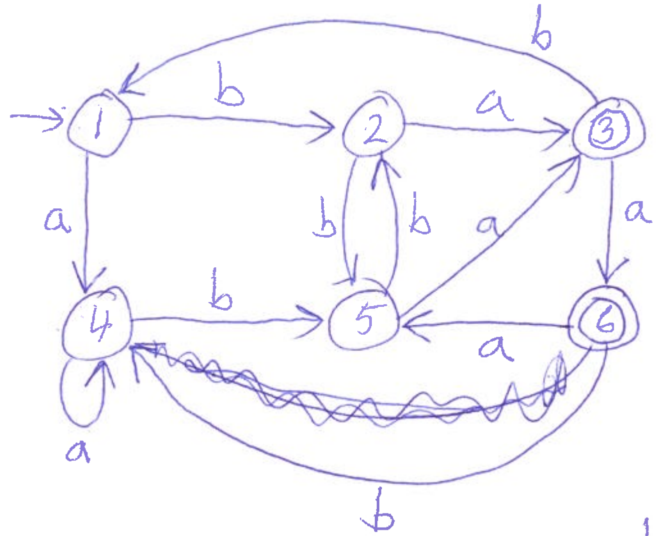
If w distinguishes t from u, then sw distinguishes q from r.

Merge 2,4,5 to get the minimum equivalent DFA:



It will be clear and ~~unab~~ unambiguous where to put arrows, start state, accepting states (no conflicts). If there is a conflict, that indicates an incorrect table ~~(X that should be~~ (omitting an X).

Ex:



2	X				
3	X	X			
4		X	X		
5	X		X	X	
6	X	X	X	X	X
	1	2	3	4	5

Merge