CSCE 355
1/8/24

Course homepage:
https://cse.sc.edu/~fenner/csce355

Theory course — Mathematical.

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Start with a simple model of computation (automata)

Inputs & outputs of computers — strings

Def: An alphabet is any nonempty finite set.
For ~~alphbt~~ alphabet Σ, the elements of Σ are called symbols (also letters, characters).

Ex: $\Sigma := \{0,1\}$ binary alphabet

$\Sigma := \{0,\dots,n-1\}$ $(n>0)$ n-ary alphabet

$(n=1)$ $\Sigma := \{0\}$ unary alphabet

$\Sigma := \{a,b,c\}$

Def: Fix an alphabet Σ. A string over Σ is a finite sequence of letters (zero or more) from Σ.

Ex: $|aabac| = 5$

Def: For any string x, use $|x|$ to denote the length of x.

Ex: aa length 2

b " 1

Identify symbols from Σ with length-1 strings.

There is a unique string of length 0 (over any alphabet). We denote it by ε (lower-case epsilon). ε is never a symbol of any alphabet.

ε length 0 (unique)

Concatenation [assume any fixed alphabet Σ]

Let x and y be strings. The concatenation of x and y (or x followed by y) is the string starting with x and continuing with y. Denoted xy.

Ex: $x := aaba$, $y := cb$; $xy = aabacb$, $yx = cbaaba$ different

Concat is associative: $(xy)z = x(yz) = xyz$

generally: $x_1 x_2 \cdots x_n$ concat of $x_1, x_2, \ldots, x_n$

Special case: $x^n := \underbrace{xx\cdots x}_{n \text{ times}}$ $(n = 0, 1, 2, \ldots)$

Ex:

$$x^2 = xx$$
$$x^3 = xxx$$
$$x^1 = x$$
$$x^0 = \varepsilon$$

Note: $x\varepsilon = x = \varepsilon x$ for any string x

Note $|xy| = |x| + |y|$ ″ ″ ″ ″ x, y

Proofs by induction on string length.

Basic fact: For any string x, exactly one of the following holds:

1. $x = \varepsilon$ or
2. There exist unique string y and unique symbol a such that $x = ya$

In case 2, $|y| = |x| - 1$ and
y is called the principal prefix of x
a is called the last symbol of x.

Induction on string length:

$x = \varepsilon$ usually the base case:
prove that ε satisfies what you want to prove.

$x = ya$ is the inductive case.
~~Can use~~ Assuming what you want to prove is true for y, prove it true for x.

Conclude that every string satisfies what you want to prove.

Another use of induction: defining functions on strings: To define a function f on all strings:

base case → 1. define $f(\varepsilon)$

inductive case → 2. define $f(x)$ for $x = ya$ in terms of $f(y)$

<u>Ex</u>: length function:

$x = \varepsilon$ $\quad |\varepsilon| = 0$

$x = ya$ $\quad |ya| = |y| + 1$ $\quad$ (any $a \in \Sigma$)

<u>Def</u>: Σ^* denotes the set of all strings over Σ.

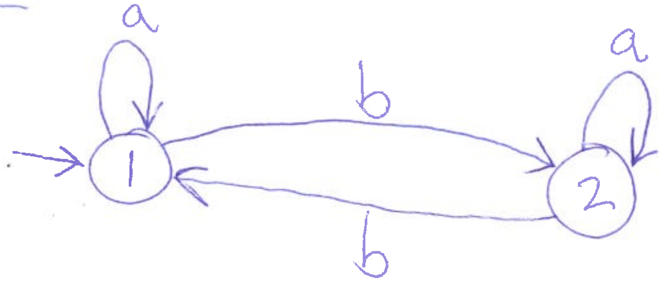
Ex: $\{0\}^* = \{\varepsilon, 0, 00, 000, \ldots, 0^n, \ldots\}$

$\{0,1\}^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, \ldots\}$

Automata

Ex: $\Sigma = \{a, b\}$

Ex:



2-state automaton

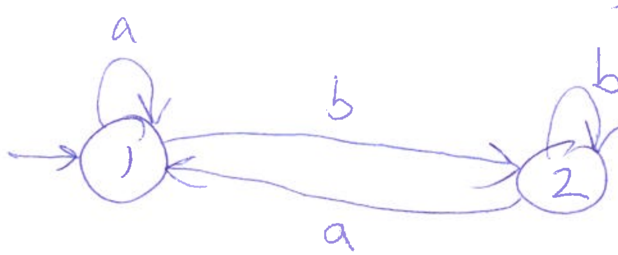
Input string: abbaba

$$\begin{array}{ccccccc} & a & b & b & a & b & a \\ 1 & 1 & 2 & 1 & 1 & 2 & 2 \end{array}$$

final state 1: even # of b's

" " 2: odd # of b's

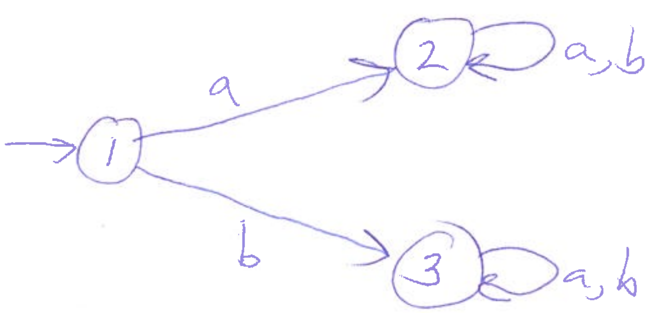
Ex:



~~final~~ final state 2: last symbol is b

" " 1: " " is not b

(either last symbol a or input is ε)

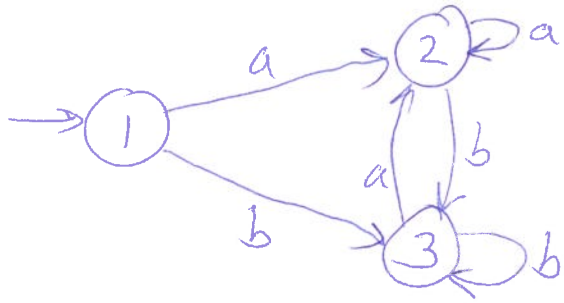


1 if ε

2 if 1st sym is a

3 if 1st sym is b

(6)



1 if ε

2 if ~~last~~ sym is a
last

3 if ~~last~~ sym is b
last