1. This exercise is adapted from Exercise 2.2.1 on pages 52–53, which is formulated somewhat vaguely. Consider the marble-rolling toy (redrawn from Figure 2.8):

A marble is dropped at A or B. Levers $x_1$, $x_2$, and $x_3$ cause the marble to fall either to the left or to the right. Whenever a marble encounters a lever, it causes the lever to reverse after the marble passes, so the next marble will take the opposite branch.

Model this toy as a finite automaton. An input to the automaton is a string over the alphabet \{A, B\}, which represents a sequence of marbles being dropped into the toy. The toy is initially in the configuration, above, before any marbles are dropped. Say that a sequence of marble drops is accepted exactly in the case that if one additional marble were to be dropped in, it would go out through D regardless of where it was dropped.

2. Recall the definition of the extended transition function $\hat{\delta}$ of a DFA $(Q, \Sigma, \delta, q_0, F)$, which takes as input any state $q \in Q$ and any string $w \in \Sigma^*$ and outputs the state reached after reading $w$ when starting in $q$. It is defined inductively (recursively) as follows:

**Basis:** For every state $q \in Q$, $\hat{\delta}(q, \varepsilon) = q$. 
INDUCTION: For state $q \in Q$, every string $x \in \Sigma^*$, and every symbol $a \in \Sigma$,
\[
\hat{\delta}(q, xa) = \delta(\hat{\delta}(q, x), a).
\]
(See Section 2.2.4 of the text.)

(a) Let $\hat{\delta}$ be the extended transition function for the DFA $A$ in Problem 7 of Assignment 3. Find $\hat{\delta}(q_1, 00)$, $\hat{\delta}(q_2, 101001)$, and $\hat{\delta}(q_0, 10011)$.

(b) Let $\hat{\delta}$ be the extended transition function of an arbitrary DFA $B = (Q, \Sigma, \delta, q_0, F)$. Prove that for any state $q \in Q$ and any strings $x, y \in \Sigma^*$,
\[
\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y).
\]
(I stated this theorem in class but did not prove it.) [Hint: Use induction on $|y|$. This is Exercise 2.2.2 on page 53. It is starred, so there is a solution on the book’s website.]

3. Describe a DFA $B$ that accepts a string over the alphabet $\{a, b, c\}$ iff its first and last symbols are different.

4. (This is adapted from Exercise 2.3.2.) Consider the following NFA:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow p$</td>
<td>${q, s}$</td>
<td>${q}$</td>
</tr>
<tr>
<td>*q</td>
<td>${r}$</td>
<td>${q, r}$</td>
</tr>
<tr>
<td>r</td>
<td>${s}$</td>
<td>${p}$</td>
</tr>
<tr>
<td>*s</td>
<td>$\emptyset$</td>
<td>${p}$</td>
</tr>
</tbody>
</table>

(a) Give the state set and set of final states in set notation.
(b) Draw a transition diagram for the NFA.
(c) Using the subset construction for the NFA.

5. Do Exercise 2.3.3 on page 66.

6. Do Exercise 2.3.4(a,b,c) on pages 66–67. Take advantage of nondeterminism as much as you can.