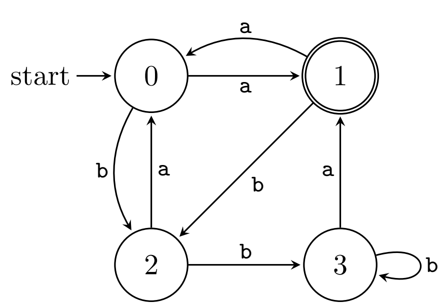
CSCE 355, Homework 2

NOTE the following definitions:

- The *binary alphabet* is the set $\{0, 1\}$.
- A *binary string* is any string over the binary alphabet.
- If w is any string, then w^R (the *reversal* of w) is w written backwards, that is, comprising the symbols of w in reverse order.
- A string x is a *prefix* of a string y iff there exists a string z such that $y = xz$.
- A string x is a *suffix* of a string y iff there exists a string z such that $y = zx$.
- A string x is a *substring* of a string y if there exist strings w, z such that $y = wxz$.

Typesetting convention: Lowercase letters that are formal symbols in a formal alphabet such as Σ will be rendered in `typewriter` font, a fixed-pitch font similar to Courier. Lowercase letters from near the beginning of the Roman alphabet set in math italic (e.g., a, b, c) will stand for variables that can take on formal symbols from Σ as values, and lowercase letters from near the end of the Roman alphabet set in math italic (e.g., w, x, y, z) will stand for variables that can take on strings in Σ^* as values. These variables will always be quantified, so there should be no ambiguity. For example, in the statement, "For all $x \in \Sigma^*$ and $a \in \Sigma$, $xa \in \Sigma^*$", x can be any string over Σ, a can be any symbol in Σ, and xa denotes the concatenation of x with a.

1. Consider the following DFA:



   (a) For each of the strings below, say which state the DFA is in after reading the string, and say whether or not the DFA accepts the string.

   `aaa` `bb` `bbb` `abab` `bbbbbbbbbbbbbbbaaa` ε `aabbbbababbaaabbaabbababbbb`

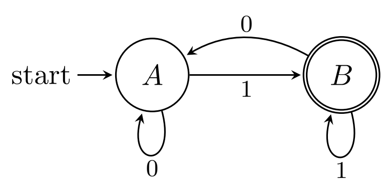
(b) Give two different strings of length 4 that each make the DFA go from state 0 to state 1.

2. Draw a DFA with alphabet $\{0,1\}$ that accepts a binary string x iff x has odd length, i.e., iff $|x|$ is odd.

3. Let A be the DFA given by the following tabular form:

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

(A accepts a binary string iff it represents a multiple of 3.) Recall the DFA described in class (here we'll call it B) that accepts a binary string iff the string ends with `a`. Here is the same one but with the binary alphabet:



Recall the product construction from class. Draw the diagram for the product of A and B so the resulting DFA recognizes the language $L(A) \cap L(B)$.

4. Describe a DFA B that accepts a string over the alphabet $\{\texttt{a}, \texttt{b}, \texttt{c}\}$ iff its first and last symbols are different.

5. Consider the following two languages over the alphabet $\{\texttt{a}, \texttt{b}\}$:

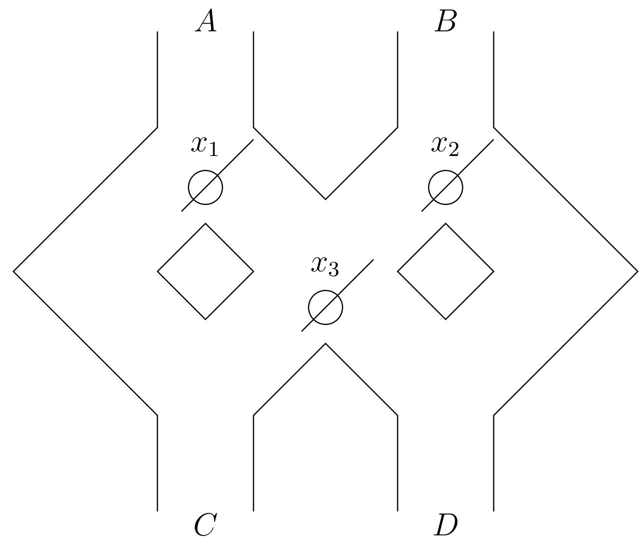
$$L_1 = \{w \mid w \text{ is either the empty string or ends with } \texttt{b}\} ,$$
$$L_2 = \{w \mid \text{there is a } \texttt{b} \text{ followed by an } \texttt{a} \text{ somewhere in } w\} .$$

(a) Draw a 2-state DFA recognizing L_1 and a 3-state DFA recognizing L_2.

(b) Using your answer and the product construction, draw a DFA recognizing $L_1 \cap L_2$. Do *not* perform any optimizations (e.g., removing unreachable states or transitions, or merging indistinguishable states).

6. Give the transition diagram for a DFA over the alphabet $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}\}$ that accepts a string w iff w contains `ab` as a substring but does not contain `abb` as a substring. What is the least number of states you need?

7. (Optional) This exercise is adapted from Exercise 2.2.1 on pages 52–53, which is formulated somewhat vaguely. Consider the marble-rolling toy (redrawn from Figure 2.8):



A marble is dropped at A or B. Levers x_1, x_2, and x_3 cause the marble to fall either to the left or to the right. whenever a marble encounters a lever, it causes the lever to reverse after the marble passes, so the next marble will take the opposite branch.

Model this toy as a finite automaton. An input to the atomaton is a string over the alphabet $\{A, B\}$, which represents a sequence of marbles being dropped into the toy. The toy is initially in the configuration above before any marbles are dropped (so that the first ball will exit at C regardless of where it is dropped). Say that a sequence of marble drops is *accepted* exactly in the case that if one additional marble were to be dropped in, it would go out through D regardless of where it was dropped.