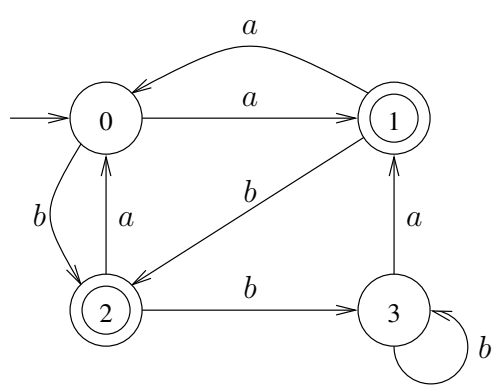
CSCE 355, Assignment 2
Answers to Selected Exercises

NOTE: I stick with the tabular form in my answers, just because transition diagrams are time-consuming to typeset. Please don't interpret what I do here as any discouragement from your giving transition diagrams in your answers.

1. Consider the following DFA:



   (a) For each of the strings below, say which state the DFA is in after reading the string, and say whether or not the DFA accepts the string.

   $aaa \qquad bb \qquad bbb \qquad abab \qquad bbbbbbbbbbbbbbaaa \qquad \varepsilon \qquad aabbbbababbaaabbaabbababbbb$

   (b) Give two different strings of length 4 that each make the DFA go from state 0 to state 1.

   Answer:

   (a)

aaa	bb	bbb	$abab$	$bbb \cdots bbbaaa$	ε	$aabbbb \cdots ababbbb$
$1, \text{acc}$	$3, \text{rej}$	$3, \text{rej}$	$2, \text{acc}$	$1, \text{acc}$	$0, \text{rej}$	$3, \text{rej}$

   (b) $abba$ and $abaa$ (also $bbba$)

2. Draw a DFA with alphabet $\{0, 1\}$ that accepts a binary string x iff x has odd length, i.e., iff $|x|$ is odd.

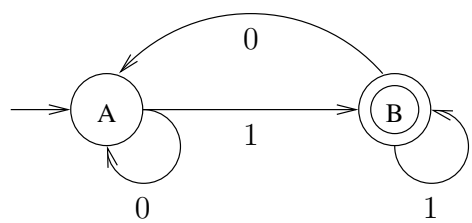
Answer: Here is the tabular form:

	0	1
$\rightarrow E$	O	O
$*O$	E	E

3. Let A be the DFA given by the following tabular form:

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

(A accepts a binary string iff it represents a multiple of 3.) Recall the DFA described in class (here we'll call it B) that accepts a binary string iff the string ends with 1:



Recall the product construction from class. Draw the diagram for the product of A and B so the resulting DFA recognizes the language $L(A) \cap L(B)$.

Answer: Here is the tabular form (note the consistent way I label and order the states):

	0	1
$\rightarrow A_0$	A_0	B_1
A_1	A_2	B_0
A_2	A_1	B_2
$*B_0$	A_0	B_1
B_1	A_2	B_0
B_2	A_1	B_2

4. Describe a DFA B that accepts a string over the alphabet $\{a, b, c\}$ iff its first and last symbols are different.

Answer:

	a	b	c
$\rightarrow 0$	A	B	C
A	A	A'	A'
$*A'$	A	A'	A'
B	B'	B	B'
$*B'$	B'	B	B'
C	C'	C'	C
$*C'$	C'	C'	C

5. Consider the following two languages over the alphabet $\{a, b\}$:

$$L_1 = \{w \mid w \text{ is either the empty string or ends with } b\} ,$$
$$L_2 = \{w \mid \text{there is a } b \text{ followed by an } a \text{ somewhere in } w\} .$$

(a) Draw a 2-state DFA recognizing L_1 and a 3-state DFA recognizing L_2.

(b) Using your answer and the product construction, draw a DFA recognizing $L_1 \cap L_2$. Do *not* perform any optimizations (e.g., removing unreachable states or transitions, or merging indistinguishable states).

Answer:

(a)

	a	b
$\rightarrow *X$	Y	X
Y	Y	X

	a	b
$\rightarrow 0$	0	1
1	2	1
$*2$	2	2

(b)

	a	b
$\rightarrow X0$	$Y0$	$X1$
$X1$	$Y2$	$X1$
$*X2$	$Y2$	$X2$
$Y0$	$Y0$	$X1$
$Y1$	$Y2$	$X1$
$Y2$	$Y2$	$X2$

or

	a	b
$\rightarrow 0X$	$0Y$	$1X$
$0Y$	$0Y$	$1X$
$1X$	$2Y$	$1X$
$1Y$	$2Y$	$1X$
$*2X$	$2Y$	$2X$
$2Y$	$2Y$	$2X$

6. Give the transition diagram for a DFA over the alphabet $\Sigma = \{a, b, c\}$ that accepts a string w iff w contains ab as a substring but does not contain abb as a substring. What is the least number of states you need?

Answer: Here are DFAs for the former and latter languages, respectively:

	a	b	c
$\rightarrow X$	Y	X	X
Y	Y	Z	X
$*Z$	Z	Z	Z

	a	b	c
$\rightarrow *0$	1	0	0
$*1$	1	2	0
$*2$	1	3	0
3	3	3	3

Using the product construction, we build the unoptimized (12-state) product DFA for the

answer:

	a	b	c
$\rightarrow X0$	$Y1$	$X0$	$X0$
$X1$	$Y1$	$X2$	$X0$
$X2$	$Y1$	$X3$	$X0$
$X3$	$Y3$	$X3$	$X3$
$Y0$	$Y1$	$Z0$	$X0$
$Y1$	$Y1$	$Z2$	$X0$
$Y2$	$Y1$	$Z3$	$X0$
$Y3$	$Y3$	$Z3$	$X3$
$*Z0$	$Z1$	$Z0$	$Z0$
$*Z1$	$Z1$	$Z2$	$Z0$
$*Z2$	$Z1$	$Z3$	$Z0$
$Z3$	$Z3$	$Z3$	$Z3$

States $X1, X2, X3, Y0, Y2, Y3$ are unreachable from $X0$ and can be eliminated to obtain the following 6-state sane DFA:

	a	b	c
$\rightarrow X0$	$Y1$	$X0$	$X0$
$Y1$	$Y1$	$Z2$	$X0$
$*Z0$	$Z1$	$Z0$	$Z0$
$*Z1$	$Z1$	$Z2$	$Z0$
$*Z2$	$Z1$	$Z3$	$Z0$
$Z3$	$Z3$	$Z3$	$Z3$

This DFA is minimal (all pairs of states are distinguishable) and alone suffices for an answer.