\[ x = \log x \quad \text{meaning } x = 2^\log x \]

\[ x = \log_2 x \quad \text{meaning } x = \log\text{binary } \log x \]

In \( x \) means natural logarithm

\[ x = \log e \quad \text{meaning } x = \log\text{natural } \log x \]

\section*{Notation}

But you should know enough to know the difference.

Sometimes in the narrower context of computer science one can be sloppy and still be correct.

Computer scientists are often very sloppy and not rigorous.

These are terms that should be defined with complete mathematical rigor.

\section*{2.1 Some Initial Notations

Asymptotic Notations}
we should use the definition as a set and we should write more pedantically definition of \((x)f\) as the set of all functions with growth rates as above. Technically, this is the notion of the asymptotic growth of a given function. The other is the needs in analysis of functions. One is the notion of the asymptotic growth of a given function. The other is the

\[ ((x)f)O \in (x)f' \]

Stoppage 2: There is also a standard abuse of notation that we shall use. There are two concepts one are asserted to be in fact be \((\cdot)O \in \text{ the rigorous sense.}\)

Stoppage 3: The text omits the absolute value symbol, This is a rare lapse in mathematical rigor, and the correct notation requires the absolute value.

\[ (x)f \cdot O \geq |(x)f| \iff \exists c < x \text{ such that } c \in f \]

Stoppage 1: The text omits the absolute value symbol, This is a rare lapse in mathematical rigor, and

\[ ((x)f)O = (x)f \]

Definition 2.1. We write

2.2 Big O (Landau, 1980s)
and this clearly goes to 0 as $x$ goes to infinity,

$$\frac{x}{1} = \frac{x + y}{y}$$

**Proof.**

$$(x + y) = x$$

**Theorem 2.3.** For any fixed $k$ and any fixed $\alpha$, we have

```
\text{Gently smaller}.
```

The $O$ notation is like a "less equal" saying that $(x)f$ is no larger than $f(x)$. The $O$ notation is

$$\infty \leftarrow x < a \quad \leftarrow \left| (x)f/(x)f \right| f$$

$$((x)f) = (x)f$$

**Definition 2.2.** We write

2.3 Little Oh (Landau, 1990s)
Introducing to infinity violate the $\gtrsim$.

**NOTE:** $O(f) \cdot \Omega(g)$ is stronger if requires all values to obey the $\gtrsim$. Classic $O(f)$ requires only that a subsequence

\[(x) g \cdot \Omega f \gtrsim \| (x) f \| \iff c < x\]

If for any fixed constant $c$, there exists a constant $C$ such that

\[(x) g \cdot \Omega f = (x) f\]

**Definition 2.5.** We write

2.5 Knuth’s Big Omega (Knuth’s corrputed notation, about 1975)

\[(x) g \cdot \Omega f \gtrsim \| (x) f \| \iff c < x\]

If for any fixed constant $c$, there exists a constant $C$ such that $x_i \rightarrow \infty$, such that for any sequence $x_i$, $x_i \cdot \cdots \cdot x_i$, $\cdots$, not $o(f)$, that is, if there exists a sequence $x_i$, $x_i \rightarrow \infty$, such that for any

\[(x) g \cdot \Omega f = (x) f\]

**Definition 2.4.** We write

2.4 Big Omega (Hardy, about 1915)
patiological graph with $n$ nodes.

even if the function $f(G,u,v)$ was only $\cup (u,v)$, the sequence forming $\cup (u,v)$, with the sequence forming $\cup (u,v)$, then we could say

The algorithm has worst case running time $\Omega(n)$. The different "nature" of the graph. If for every $u$ there exists a pathological worst case graph, then we could say

"Consider a graph algorithm with running time $f(G,u)$ depending on the number of nodes, $u$. in the sense. Also that in the context of algorithm analysis, sometimes the two notations can agree in a restricted

Note also that in the context of algorithm analysis, sometimes the two notations can agree in a restricted

sure you know which definition is being used.

lead to some confusion. When you read a book or paper that uses the notation $O(.)$, you will have to make

Most computer scientists will not know about the traditional notation. The difference between the two can

theory, from where these concepts arose.

traditional definition will be used by most mathematicians, especially those in number theory and function

Note the difference between the traditional definition of $\Omega(.)$ and the modern computation of Knuth.
\[ \infty \in \infty \quad \text{as} \quad \infty \quad \text{then} \quad (x)B / (x)f \quad \text{then} \quad (x)B \circ = (x)f \quad \text{if} \]

\[
((x)B) \circ = (x)f
\]

**Definition 2.8.** We write

2.7 Little Omega

\[
((u)B)^{\mathcal{U}} = (u)f \quad \text{and} \quad ((u)B)O = (u)f \iff ((u)B)\Theta = (u)f
\]

**Theorem 2.7.** For any two functions \((u)g\) and \((u)f\), we have

\[
(x)B \cdot \mathcal{C} \supset (x)f \supset (x)B \cdot \mathcal{C} 
\]

If there exist constants \(c, \mathcal{C}, \mathcal{C}^1\), and \(\mathcal{C}\), such that

\[
((x)B)\Theta = (x)f
\]

**Definition 2.6.** We write

2.6 Theorem (??)
Therefore

\( (\gamma x)\mathcal{O} = (x)f \)

\( \gamma x \cdot \mathcal{O} = \)

\( (I + \gamma) \cdot \gamma x \cdot |y| \max = \)

\( (I + \cdots I + I + I) \cdot \gamma x \cdot |y| \max \geq |(x)f| \)

Now if \( I \leq y \), we have

\[ |(\gamma x/I) + \cdots + \gamma x/I + x/I + |I| \cdot |\gamma x| \cdot |y| \max \geq \]

\[ |(\gamma x/0y) + \cdots + \gamma x/0y + x/1-\gamma y + |uy| \cdot |\gamma x| \geq \]

\[ |((\gamma x/0y + \cdots + \gamma x/0y + x/1-\gamma y + uy) \cdot \gamma x| = |(x)f| \]

\[ \forall x \in \mathbb{R}, \text{ we have } |y| \max (I + y) = \mathcal{O} \text{ then } \mathcal{O} \cdot (x)f \cdot \mathcal{O} \subseteq |(x)f| \]

**Proof.** To do the formal proof, we need to show that there exists a constant \( c \) such that for

\( (\gamma x)\Theta = (x)f \) when \( \gamma x/0y + \cdots + \gamma x/ny = (x)f \)
\[(\frac{\gamma x}{\Theta} = (x)f) \text{ and thus } \frac{(\frac{\gamma x}{\Omega} = (x)f)}{0} \geq \]

\[\frac{\gamma x \cdot \mathcal{O}}{((I + u)/u - I) \cdot |u^D| \cdot |\gamma x| \geq}

\[\frac{((I + u)/|u^D| - \cdots (I + u)/|u^D| - (I + u)/|u^D| - |u^D|) \cdot |\gamma x| \geq |(x)f|}{\text{then we will have}}

\[\frac{|u^D|/(1, |u^D|) \max (1 + u) < x \text{ if for each } i, \text{ which will yield}}{\text{we now have for each } (1 + u)/|u^D| > |x/1 - y^D| \text{ we have}}

\[\frac{|x/0^D| - \cdots |x/\gamma^D| - |x/1 - y^D| - |u^D|) \cdot |\gamma x| \geq}{\text{we have}}

\[\frac{|(x/0^D + \cdots x/\gamma^D + x/1 - y^D + u^D)|) \cdot |\gamma x| \geq}{\text{we have}}

\[\frac{|(x/0^D + \cdots x/\gamma^D + x/1 - y^D + u^D)| \cdot |\gamma x| = |(x)f|}{\text{we have}}

\[\frac{(x)f \cdot \mathcal{O} \geq |(x)f|}{\text{we have}}

\[\frac{(x)f \cdot \mathcal{O} \geq |(x)f|}{\text{we have}}

\[\frac{c \cdot (x)f < x}{\text{in the other direction, we need to show that there exists a constant } C \text{ and a constant } c \text{ such that for every}}

Theorem 2.12. Let \( i \) and \( k \) be fixed and let \( f(n) = (n+i)^k \). Then \( f(n) = \Theta(n^k) \).

Since \( k \) is fixed, \((1/2)^k\) and \(2^k\) are the constants needed for the \( \Theta \).

Proof. Very crudely, we have for \( n > 1 \) that

\[
\frac{(1/2)^k}{(n/2)^k} \leq \frac{(n+1)^k}{(2n)^k} < \frac{(2n)^k}{(n-1)^k}.
\]

Then \( f(n) = \Theta(n^k) \).

Theorem 2.11. Let \( k \) be fixed and let \( f(n) = (n+1)^k \) and \( g(n) = (n-1)^k \). Then \( f(n) = \Theta(n^k) \) and

\[ g(n) = \Theta(n^k). \]

Sometimes it can be tedious to make a sum come out to a function exactly of \( n \); we go one step too far or one step too few and come up with an \( n+1 \) or \( n-1 \). The next theorems are useful in allowing us to be a little bit sloppy about these faredpost errors.
\[ 0 = \frac{\varepsilon x^2}{-y} \quad \text{Im} \]

\[ \frac{\varepsilon x^3}{x - y \log (x - y)} \quad \text{Im} \]

\[ \frac{\varepsilon x^3}{x - y \log (x - y)} \quad \text{Im} \]

\[ \frac{\varepsilon x^3}{x - y \log (x - y)} \quad \text{Im} \]

We do this with L'Hôpital's rule by differentiating numerator and denominator.

\[ 0 = \lim_{x \to \infty} \frac{\varepsilon x}{x \cdot \log x} \]

\[ \text{Proof. What we need to show is that for any fixed } \varepsilon \text{ and } \delta, \text{ we have} \]

\[ (\varepsilon x) \circ = x \cdot \varepsilon \log \]

\[ \text{Theorem 2.13. For any } \varepsilon < 0 \text{ and for any } \delta > 0, \text{ we have} \]
To bound a function from above, use the rectangles with

\[ xp(x) \int_{1-w}^{1} \geq (\varepsilon + w) f + (\zeta + w) f + (1 + w) f + (w) f \]

Proof:

\[ \int_{1+w}^{1} \geq (\gamma + w) f \]

**Theorem 2.15.** If \( f(x) \) is a monotonically decreasing function, then we have

\[ xp(x) \int_{1-w}^{1} \geq (\zeta) f \int_{u}^{w} \geq xp(x) \int_{1+w}^{1} \]

**Theorem 2.14.** If \( f(x) \) is a monotonically increasing function, then we have

\[ xp(x) \int_{1-w}^{1} \geq (\zeta) f \int_{u}^{w} \geq xp(x) \int_{1+w}^{1} \]
(ε + w)f + (z + w)f + (1 + w)f + (w)f ≥ xρ (x)f \int_{\varepsilon + w}^{1 - w} f

The right endpoints of BULLETS E. G.

To bound from below, use the rectangles with as

(\gamma + x)f
\[ \frac{1 + \gamma}{1 - \gamma (1 + u)} = x p_\gamma x \int_0^\infty < \gamma^2 \int_u^{1-i} \geq x p_\gamma x \int_0^\infty = \frac{1 + \gamma}{1 + \gamma u} \]

**Proof.** By the earlier Theorem 2.14, we have

\[ (1 + \gamma u) \Theta = < \gamma^2 \int_u^{1-i} \]

**Theorem 2.18.**

\[ (\varepsilon^w) \Theta = \frac{\gamma}{(1 + w u) (1 + u^2)} = < \gamma^2 \int_u^{1-i} \]

**Theorem 2.17.**

\[ (\varepsilon^w) \Theta = \frac{\gamma}{1 + w u} = < \gamma^2 \int_u^{1-i} \]

**Theorem 2.16.**
\[ x \log_{q} y = \log_{q} x \]

**Proof.** All logs are the same. We observe simply that

\[ (x^{a})^{b} = x^{ab} \]

**Theorem 2.21. For any fixed values of \( a \) and \( 0 < q \), \( 0 < q < 1 \), \( \log_{q} x \) is a decreasing function. Therefore**

\[ (1 - w) \log_{q} x - (u) \log_{q} x \]

**Proof.** We note that \( x / y \) is a decreasing function. Therefore

\[ (1 - w) \log_{q} x - (u) \log_{q} x \]
Introduction that it isn't really possible to back up one step and do it right. Sometimes being sloppy can cause confusion. None of these notations require so much change the meaning. Sometimes being sloppy can be careful with these notations. Sometimes being sloppy turns out to be ok. Sometimes being sloppy can

\( (u)\tilde{b} < (u)f_{\tilde{S}_n}, ((u)\tilde{b})m = (u)f \)

\( (u)\tilde{b} > (u)f_{\tilde{S}_n}, ((u)\tilde{b})o = (u)f \)

\( (u)\tilde{b} = (u)f_{\tilde{S}_n}, ((u)\tilde{b})\Theta = (u)f \)

\( (u)\tilde{b} \geq (u)f_{\tilde{S}_n}, ((u)\tilde{b})\mathcal{U} = (u)f \)

\( (u)\tilde{b} \text{ is not less than } (u)f_{\tilde{S}_n}, ((u)\tilde{b})\mathcal{U} = (u)f \)

\( (u)\tilde{b} \geq (u)f_{\tilde{S}_n}, ((u)\tilde{b})O = (u)f \)
Take note of your audience.

Infinity many positive integers does.

Real time does not have to be true for a set of points going off to infinity, but something that is true for algorithms and functions defined on the entire real line. Sometimes that is true infinitely often on the value symbols are definitely not needed.

Make sure functions are always positive (like functions that count steps in algorithms), so that the absolute
\cdot ((uf) o = (uf) \Theta ) \quad \text{and} \quad ((uf) \Theta ) = (uf) \quad (1)\\
\cdot ((uf) \Theta ) = (uf) \quad \text{and} \quad ((uf) \Theta ) = (uf) \quad (2)\\
\cdot ((uf) \Theta ) = (uf) \quad \text{and} \quad ((uf) \Theta ) = (uf) \quad (3)\\
\cdot ((uf) \Theta ) = (uf) \\
\cdot ((uf) \Theta ) = (uf) \\
\cdot ((uf) \Theta ) = (uf) \\
\cdot ((uf) \Theta ) = (uf) \\
\cdot ((uf) \Theta ) = (uf) \\
\cdot ((uf) \Theta ) = (uf) \\

The following are real.

Provided

2.10 Some simplifications and some admissions.
\{ I \mid u_{(1)}^I : 0 < I \} \min = u_{(1)}^I

less than 1, if \( I \) is \( I \) times, then

Definition 2.22. The Iterated Logarithm to be the number of \( I \) operations necessary to get something

\[ \frac{u_2}{I} > u_0 > \frac{1 + u_2}{I} \]

Note where the "weight" of the approximation is. It is in the \( u_0^I(\alpha) \) part. The rest is rather unimportant.

where

\[ u_0 \in \left( \frac{\theta}{u} \right) u_{\text{new}}^I = iu \]

In fact

\[ \left( \left( \frac{u}{I} + I \right) u \left( \frac{\theta}{u} \right) u_{\text{new}}^I = iu \right. \]

Sternberg's formula

2.11 Sternberg's formula
\[
(1-u^x \hat{y} - u^x)y = u^x \hat{y} - 1+u^x
\]
\[
(1-u^x \hat{y} - u^x)\hat{y} = u^x \hat{y} - 1+u^x
\]

so that

\[
q = \hat{y} \cdot Iy \quad \text{and} \quad \nu = \hat{y} + Iy
\]

Then since \( q \) and \( \nu \) be the roots of the quadratic equation

\[
0 = q - \nu \rho - \hat{y}
\]

\[
1-u^x(\nu \rho + q) + (1-u^x \nu - u^x)(\nu - \rho) = u^x \nu - 1+u^x
\]

What we really have, for any \( q \), is

\[
1_x = \frac{1}{x} = 0_x = q = \nu
\]

This is called a second order recurrence relation. Note that the Fibonacci numbers are defined by

\[
1-u^x \cdot q + u^x \cdot \nu = 1+u^x
\]

Let's assume that we have initial values \( u^x \nu^0, \nu, q, x^0 \), and \( x \), and subsequent values defined by

A final comment on orders of magnitude.
\[
\left(\frac{\frac{2}{s} + 1}{2} - \gamma\right) \left(\frac{\frac{2}{s} + 1}{2} + 1 - \gamma\right) = 1 - \gamma - \gamma^2
\]

**Example:** In the case of the Fibonacci numbers, the quadratic equation is

\[
\frac{(1, y - \gamma y)}{\mu y(0, x \gamma y - 1, x) - \gamma y(0, x \mu y - 1, x)} = u \cdot x
\]

Then if \( y \neq 1 \), we have

\[
\mu y(0, x \gamma y - 1, x) - \gamma y(0, x \mu y - 1, x) = u \cdot x(1, y - \gamma y)
\]

Subtract these two to get

\[
(0, x \gamma y - 1, x) \mu y = u \cdot x \gamma y - 1 + u \cdot x
\]

\[
(0, x \mu y - 1, x) \gamma y = u \cdot x \mu y - 1 + u \cdot x
\]

We unwind this recurrence.
themselves grow exponentially.

we have a $y$ main term that is dominant and a $y^2$ term that dies out. And since $y_1$ is bigger than $1$, the $x^n$

$$\frac{(1y - \tilde{y}y)}{u y (0x \tilde{y} - 1x) - \tilde{y} y (0x y - 1x)} = u x$$

So if $u$ and $\tilde{y}$ are both positive, then in the expression

we note that if $\tilde{y}$ is at least 1, then since $y_1$ is less than 1, we must have that $y_1$ is greater than 1.

and therefore the quotient of the two values goes off to infinity as we power up.

$$1 < \frac{\tilde{y}}{q \bar{A} + \tilde{p} \bar{A} + \bar{p} + \bar{I}}$$

Now, since we assume $q$ and $p$ to be positive integers, this quotient in absolute value is

$$\frac{\tilde{y}}{q \bar{A} + \tilde{p} \bar{A} + \bar{p} + \bar{I}} = \frac{q \bar{A} - \tilde{p} \bar{A} - \bar{p}}{q \bar{A} + \tilde{p} \bar{A} + \bar{p} + \bar{I}}$$

$$= \frac{q \bar{A} - \tilde{p} \bar{A} - \bar{p}}{q \bar{A} + \tilde{p} \bar{A} + \bar{p} + \bar{I}} = \frac{q \bar{A} + \tilde{p} \bar{A} - \bar{p}}{q \bar{A} + \tilde{p} \bar{A} + \bar{p} + \bar{I}}$$

$A_{II}$ the weight is in the $y_1$ term, because

$$\frac{\tilde{y}}{q \bar{A} + \tilde{p} \bar{A} - \bar{p}} = \tilde{y} < \frac{\tilde{y}}{q \bar{A} + \tilde{p} \bar{A} + \bar{p}} = \bar{y}$$

in the expression above comes from only the first term. This is because we have VLOG

Remark: if $u$ and $q$ are positive integers, as they usually are in algorithmic analysis, then all the “weight”
Then we have \( x^n \) grows at least as fast as \( x \),

\[
(1 - \nu)x(q + v) < 1 - \nu x q + \nu x v = 1 + \nu x
\]

Then the sequence \( x^n \) is positive and monotonically increasing. Since we have

Assume that \( q, x_0, x_1 \) are positive integers, as they usually will be in algorithmic analyses.

One last comment.