OPTIMAL SAMPLING METHODOLOGIES FOR HIGH-RATE STRUCTURAL TWINNING

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High-rate Dynamics in the Real-world

Civil Structures Exposed to blast

Samali, B., et al., *Review of the basics of state of the art of blast loading. Asian Journal of Civil Engineering.* (2018).

Automotive safety systems against Collision

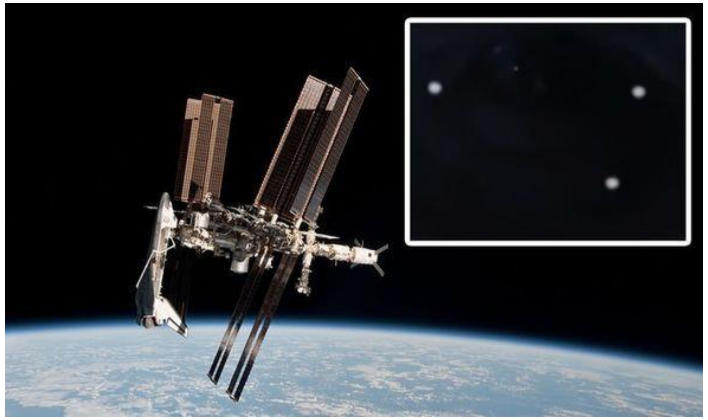
airbag deployment

Space shuttle and Aerial Vehicles Prone to In-Flight Anomalies

Hypersonic vehicles

Ballistic packages

Debris approaching space shuttle

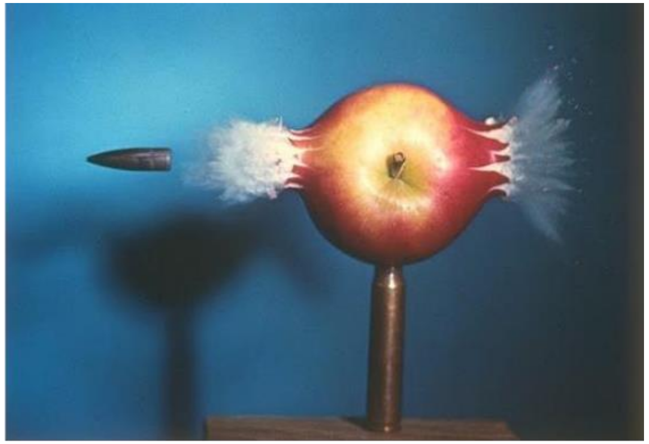


Lightning strikes on aircraft

Fighter jets

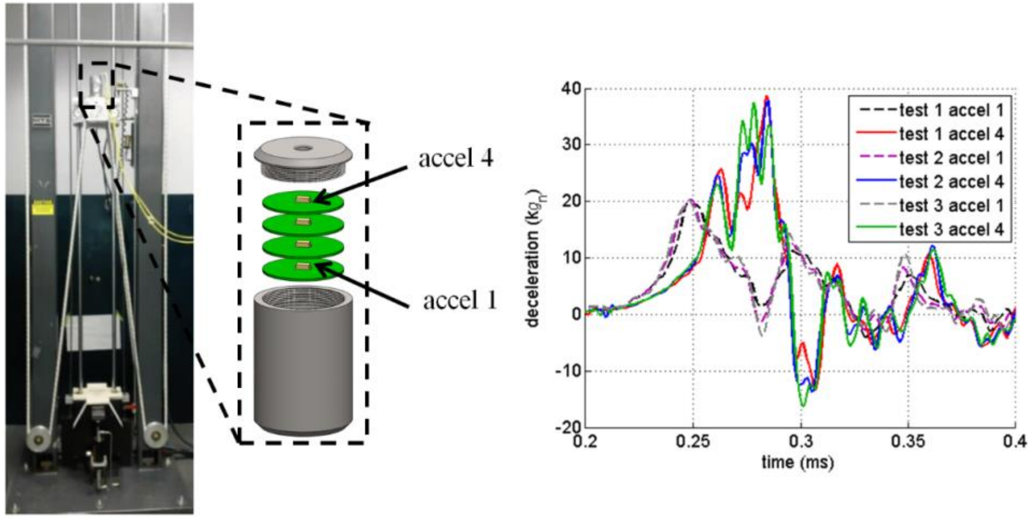
Description of High-rate Dynamics

High-rate (<100ms)



High-amplitude (acceleration > 100 *g*)

The deceleration event in drop tower tests typically lasts for 0.5ms



- Large uncertainties in the external loads.
- High levels of nonstationarity and heavy disturbance.
- Generations of unmodeled dynamics from changes in mechanical configuration.

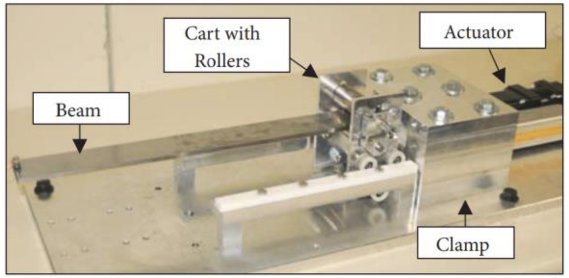
GitHub

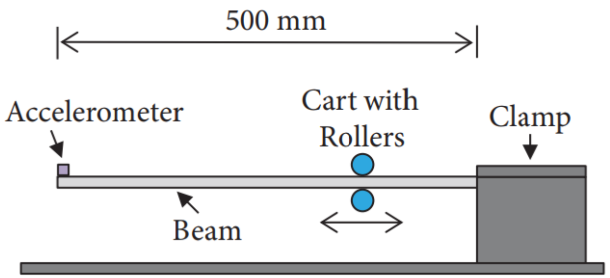
Hong, J. et al,. *Introduction to state estimation of high-rate system dynamics*. Sensors, 18(2):217, Jan 2018

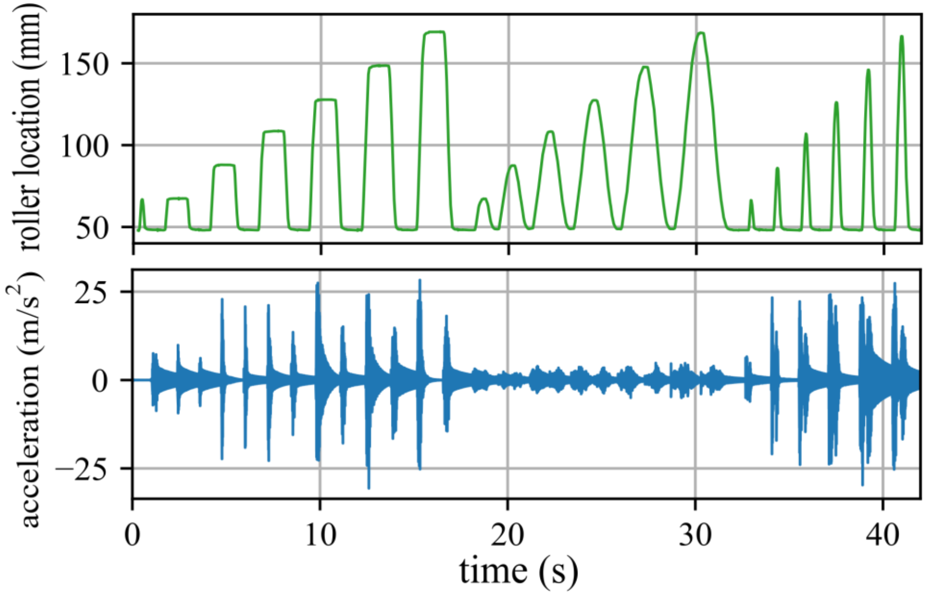
Reproducing High-rate Dynamics in Laboratory

DROPBEAR experimental testbed:

- The Dynamic Reproduction of Projectiles in Ballistic Environments for Advanced Research (DROPBEAR) was used to generate the experimental data in this work.
- Cantilever beam with a controllable roller to alter the state.
- Acceleration and pin location are recorded.
- Dataset available on GitHub at: https://github.com/High-Rate-SHM-Working-Group/Dataset-2-DROPBEAR-Acceleration-vs-Roller-Displacement

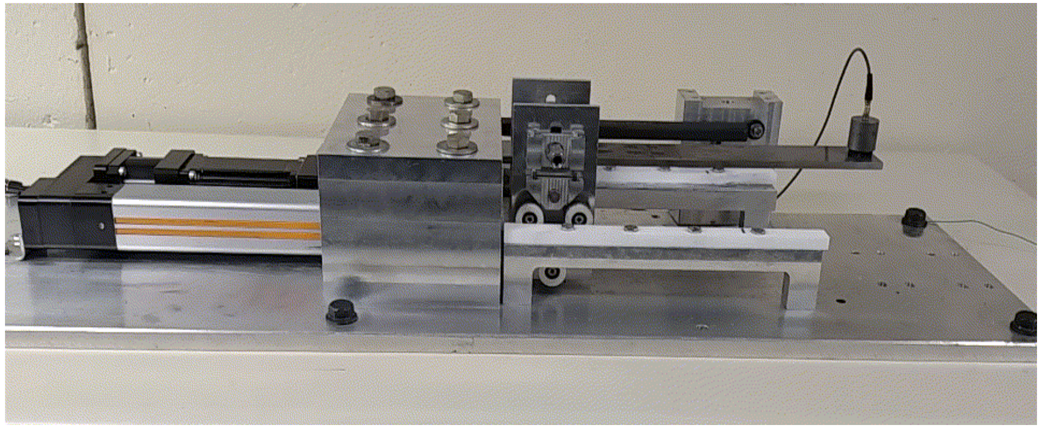


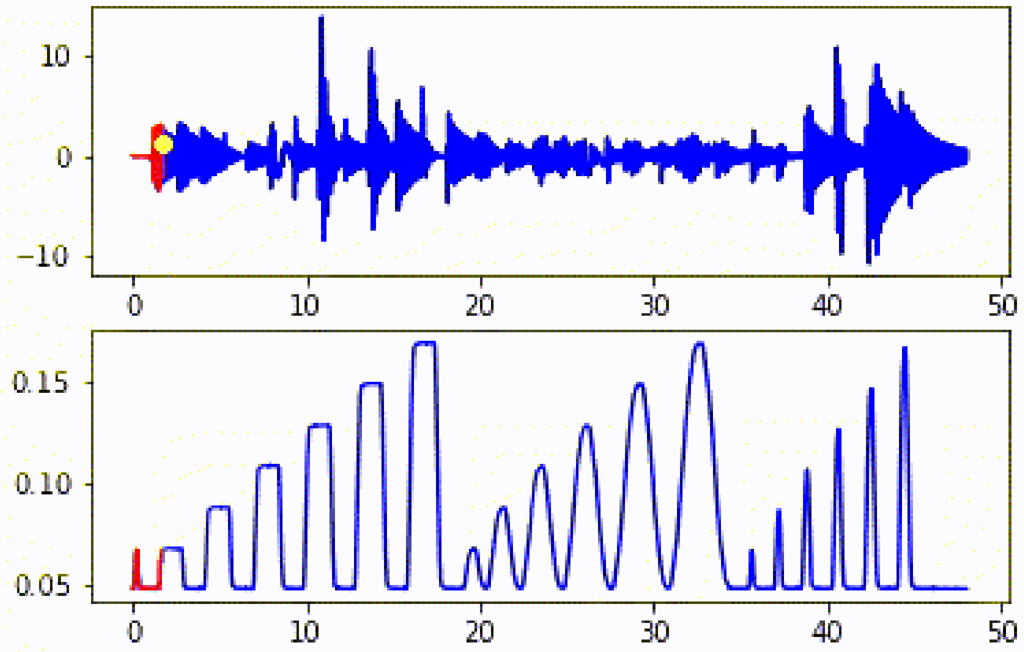




Joyce, B., Dodson, J., Laflamme, S., & Hong, J. *An experimental test bed for developing high-rate structural health monitoring methods*. Shock and Vibration, 2018.

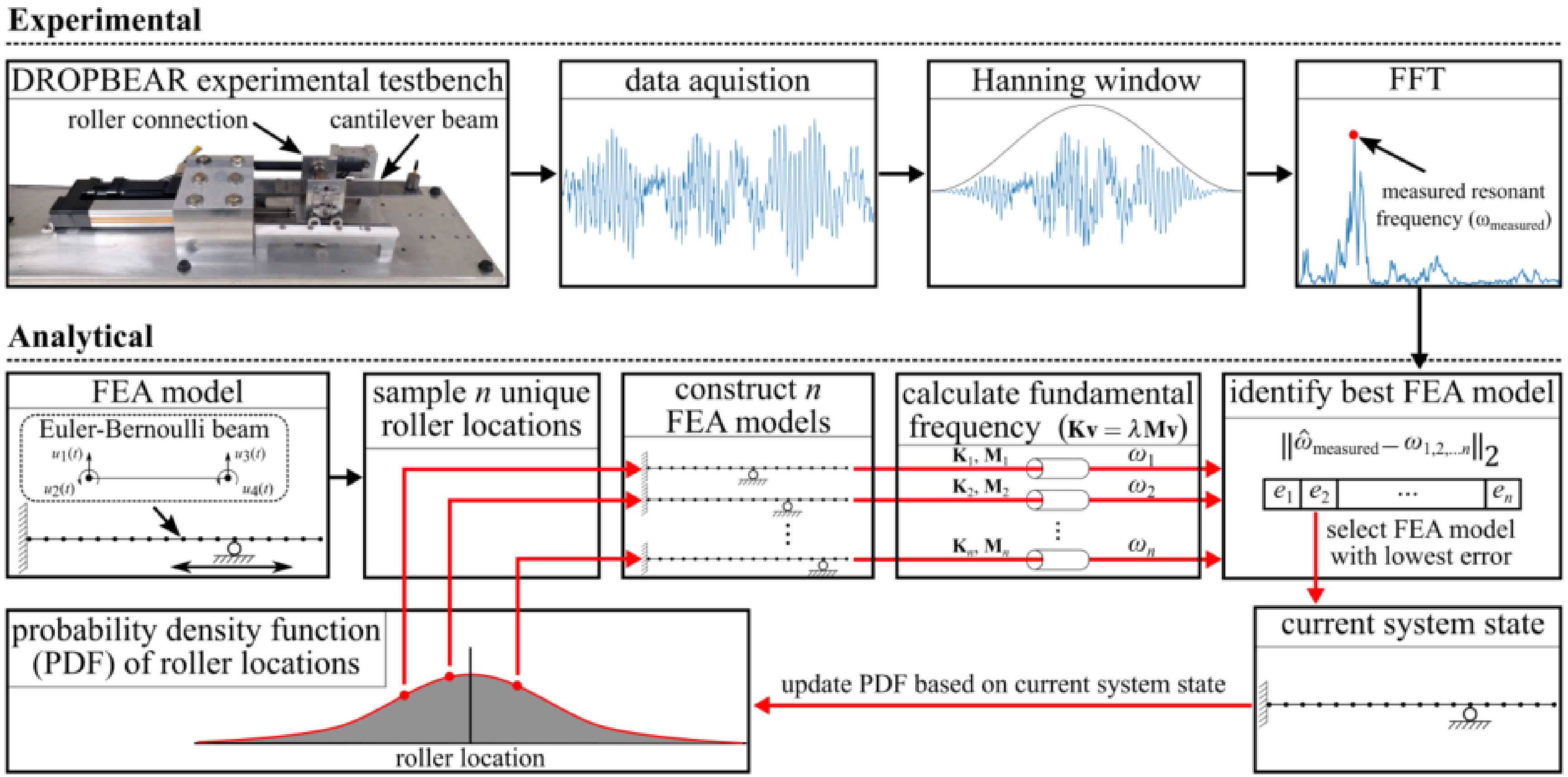
Experimental System used for Validation





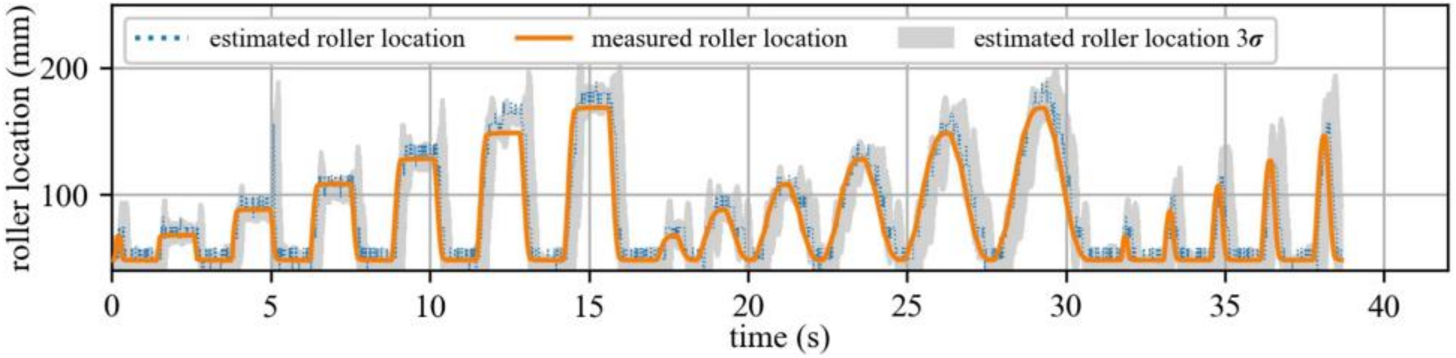
Joyce, B., Dodson, J., Laflamme, S., & Hong, J. *An experimental test bed for developing high-rate structural health monitoring methods*. Shock and Vibration, 2018.

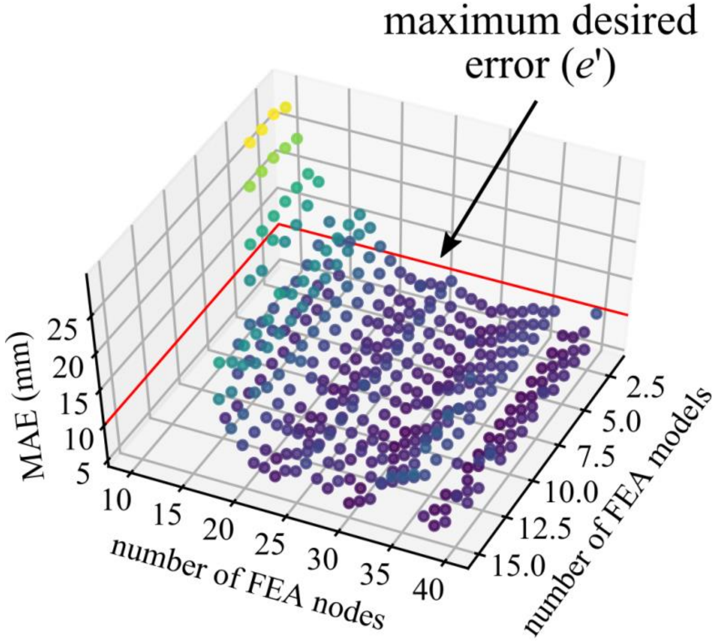
Real-time FEA model updating

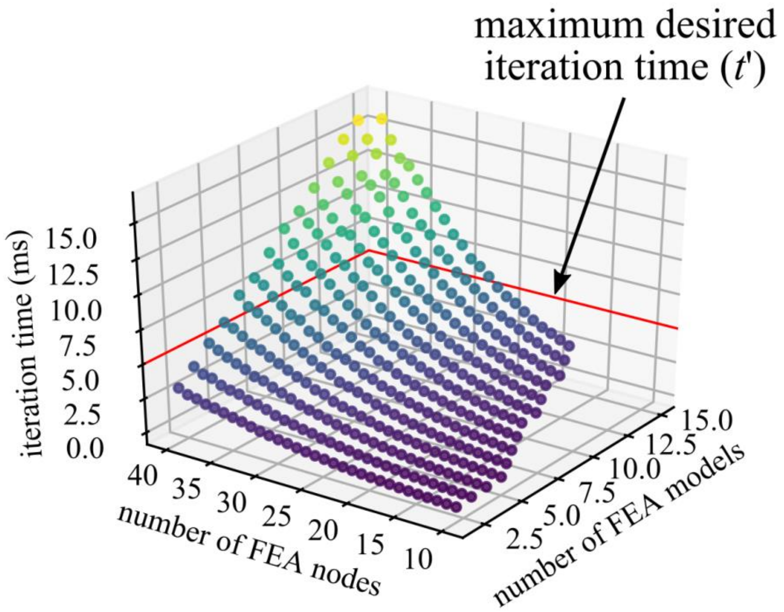


Downey A., et al,. "Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events" *Mechanical Systems and Signal Processing* ***138***, 2020

Real-time FEA model updating Results





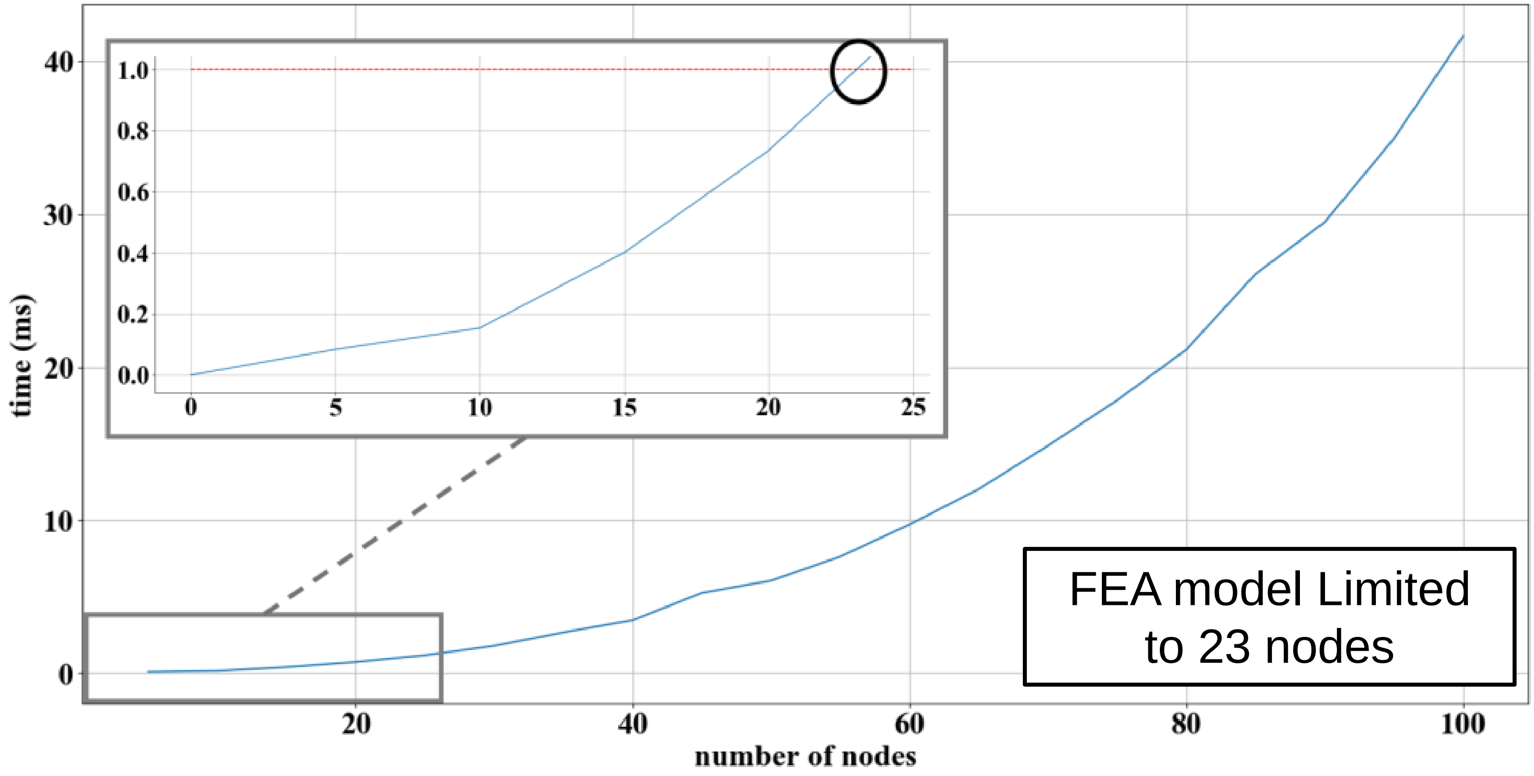


Downey A., et al,. "Millisecond Model Updating for Structures Experiencing Unmodeled High-Rate Dynamic Events" *Mechanical Systems and Signal Processing* ***138***, 2020

FEA Computation speed for the DROPBEAR

General Eigenvalue solutions accurately estimates the state of the DROPBEAR

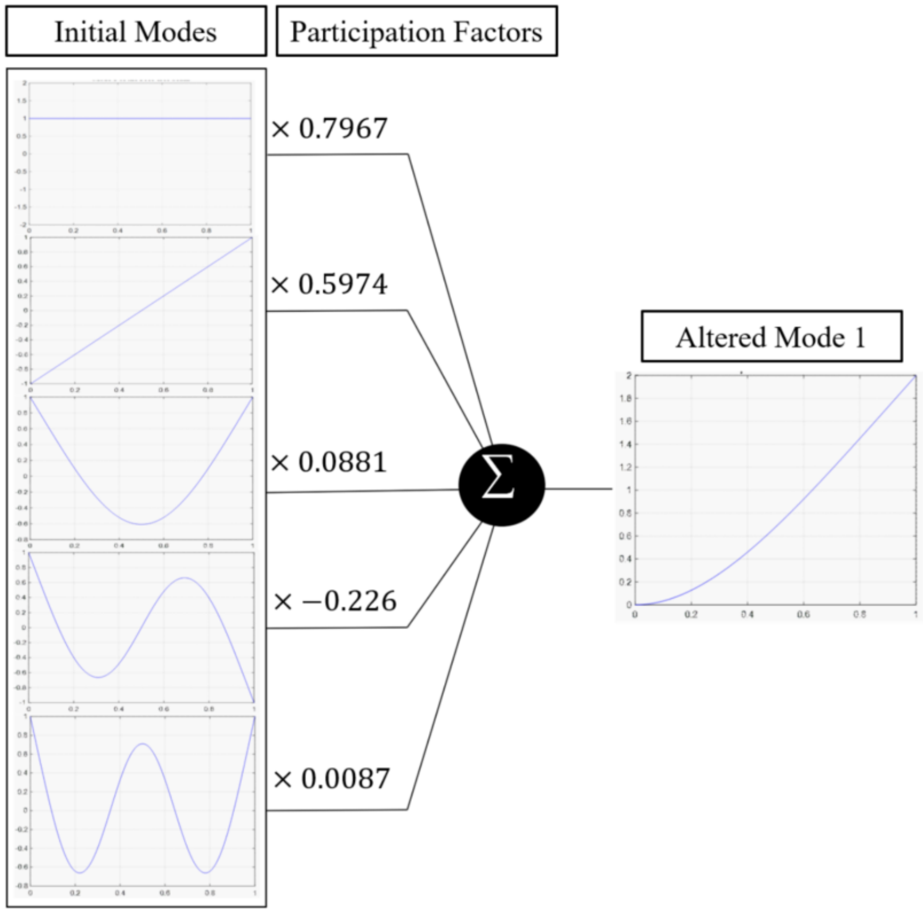
Solving for system's frequencies accounted for 90% of algorithm iteration time



Carroll, M., Downey, A., Dodson, J., Hong, J. and Scheppegrell, J., "Analysis of Computation Speeds of Eigenvalue Solutions for High-Rate Structural Health Monitoring."

Local Eigenvalue Modification Procedure (LEMP)

- Developed by Wesseinburger in 1968
- Identifies physical changes to the system such as mass, stiffness or damping using changes such as frequencies or mode shapes
- Model the altered state as a mixture of the initial state and changes made to the initial state
- Reduces the GE equation to a set of second-order equations

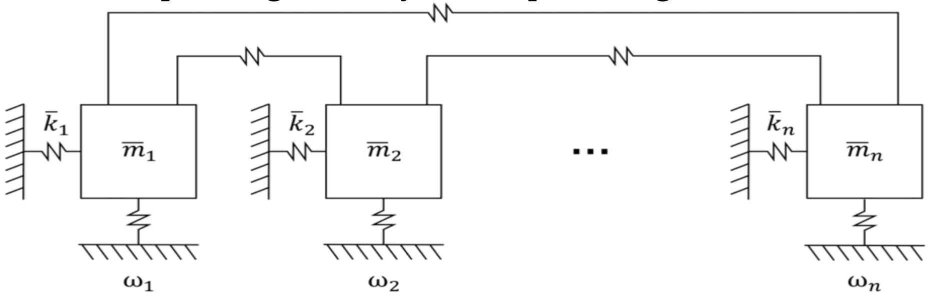


Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration,* pp. 14-25. 2003
Drnek, C. R., "*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*," (2020).

Local Eigenvalue Modification Procedure (LEMP)

n independent single DOF systems representing the initial state

Coupled single DOF systems representing the altered state

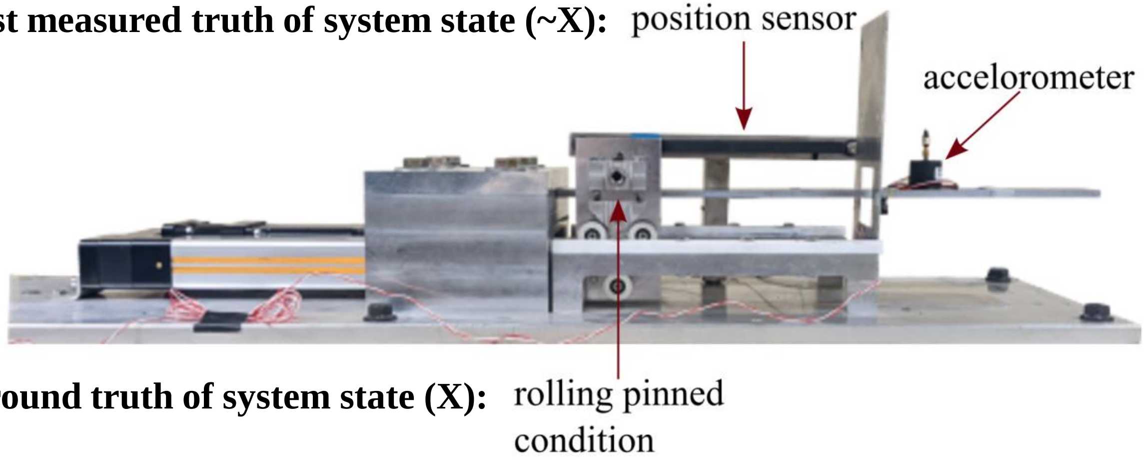


	Initial State	Modification	Altered State	
Physical Space	$[\mathbf{M}_1], [\mathbf{K}_1]$	→ $[\Delta\mathbf{M}_{12}], [\Delta\mathbf{K}_{12}]$ →	$[\mathbf{M}_2], [\mathbf{K}_2]$	*'n'* Physical DOF
Modal Transformation	$\{x\} = [U_1]\{p_1\}$	$$\frac{-1}{\alpha} = \sum_{r=1}^{m} \frac{v_r^2}{\omega_r^2 - \Omega_r^2}$$ **Solved using Divide and Conquer method**	$\{x\} = [U_2]\{p_2\}$	*m<<n*
Modal Space	$[\omega_1^2], [U_1]$	→ $\{p_1\} = [U_{12}]\{p_2\}$ →	$[\Omega_2^2], [U_2]$	*'m'* Modal DOF

Avitabile, P., "*Twenty Years of Structural Dynamic Modification- A Review*," *Sound and Vibration*, pp. 14-25. 2003
Drnek, C. R., "*Local eigenvalue modification procedure for real-time model updating of structures experiencing high-rate dynamic events*," (2020).

Current methodology

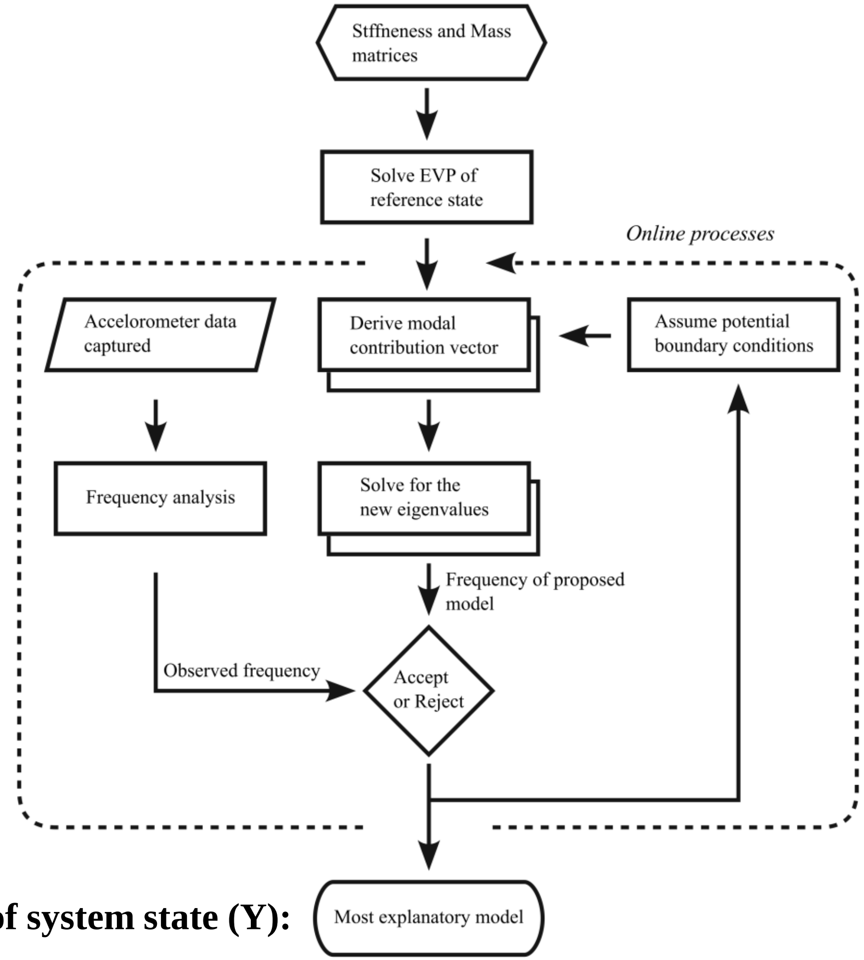
Closest measured truth of system state (~X):



Ground truth of system state (X):

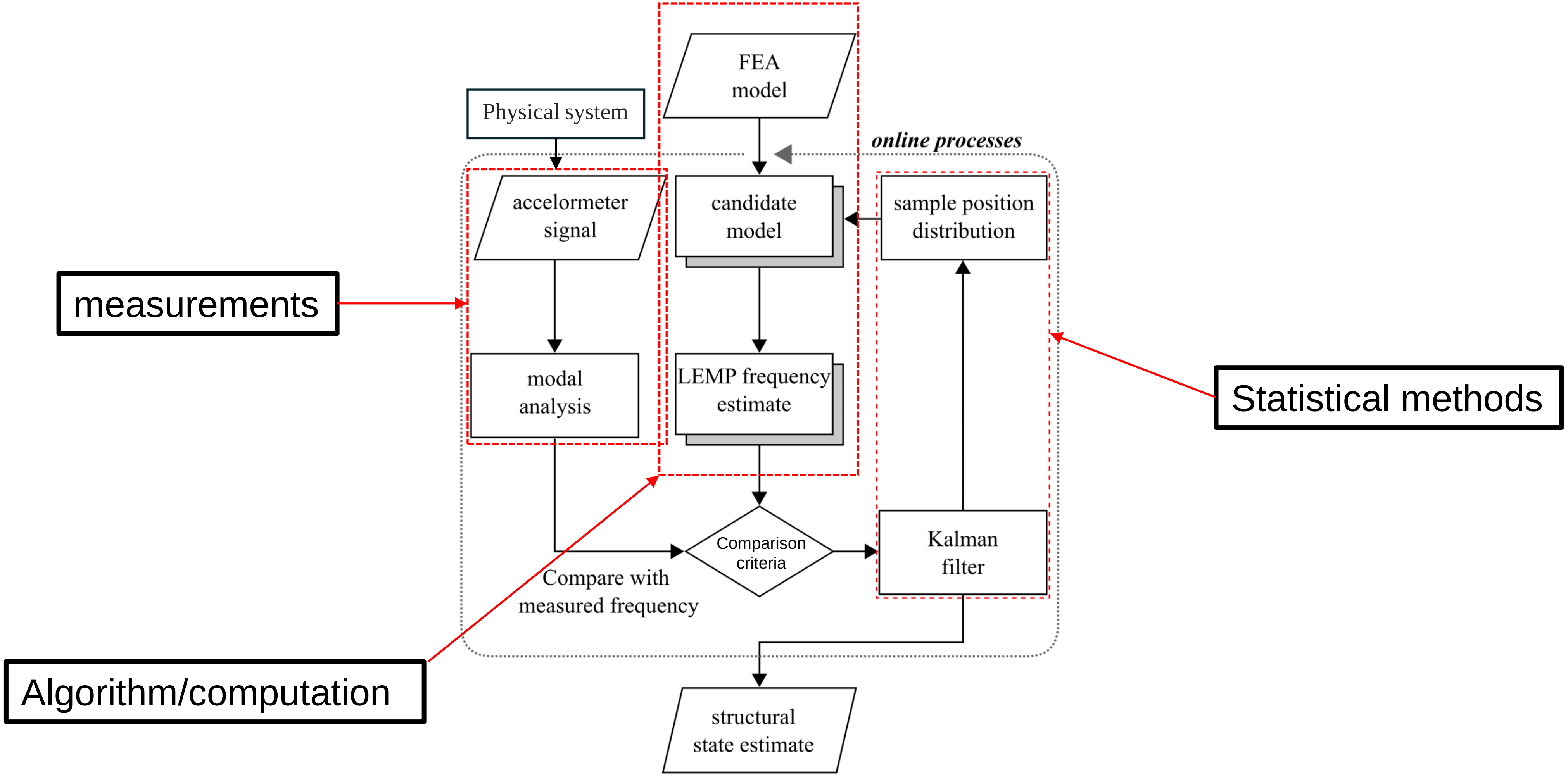
Assumed to be Constant Velocity Model:

$$\dot{x} = Ax + \Omega_p$$
$$y = Cx + \Omega_m$$



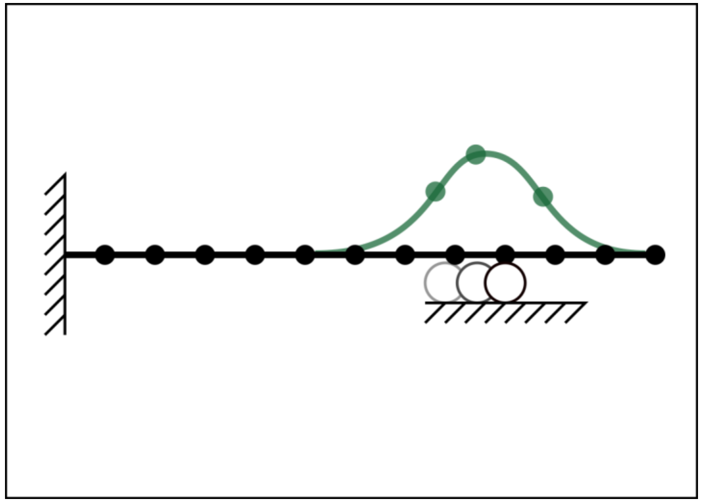
Measurement of system state (Y):

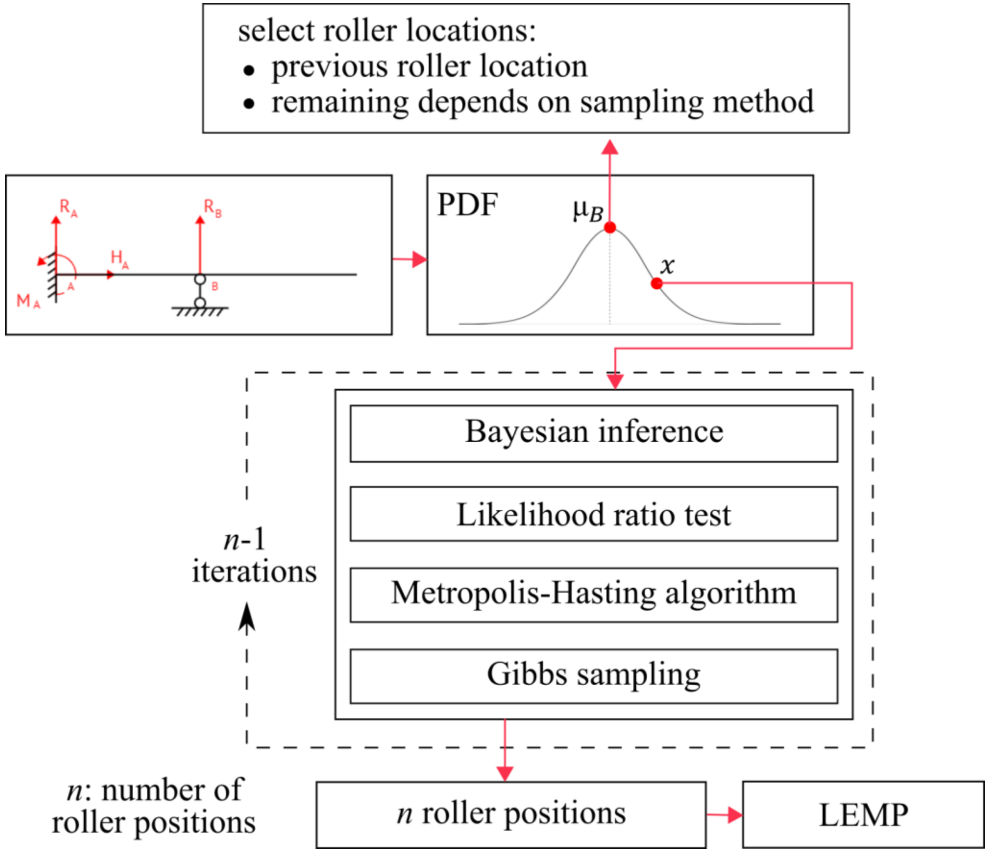
Data Fusion


FEA model
Physical system
online processes
accelormeter signal
candidate model
sample position distribution
measurements
modal analysis
LEMP frequency estimate
Statistical methods
Comparison criteria
Kalman filter
Compare with measured frequency
Algorithm/computation
structural state estimate

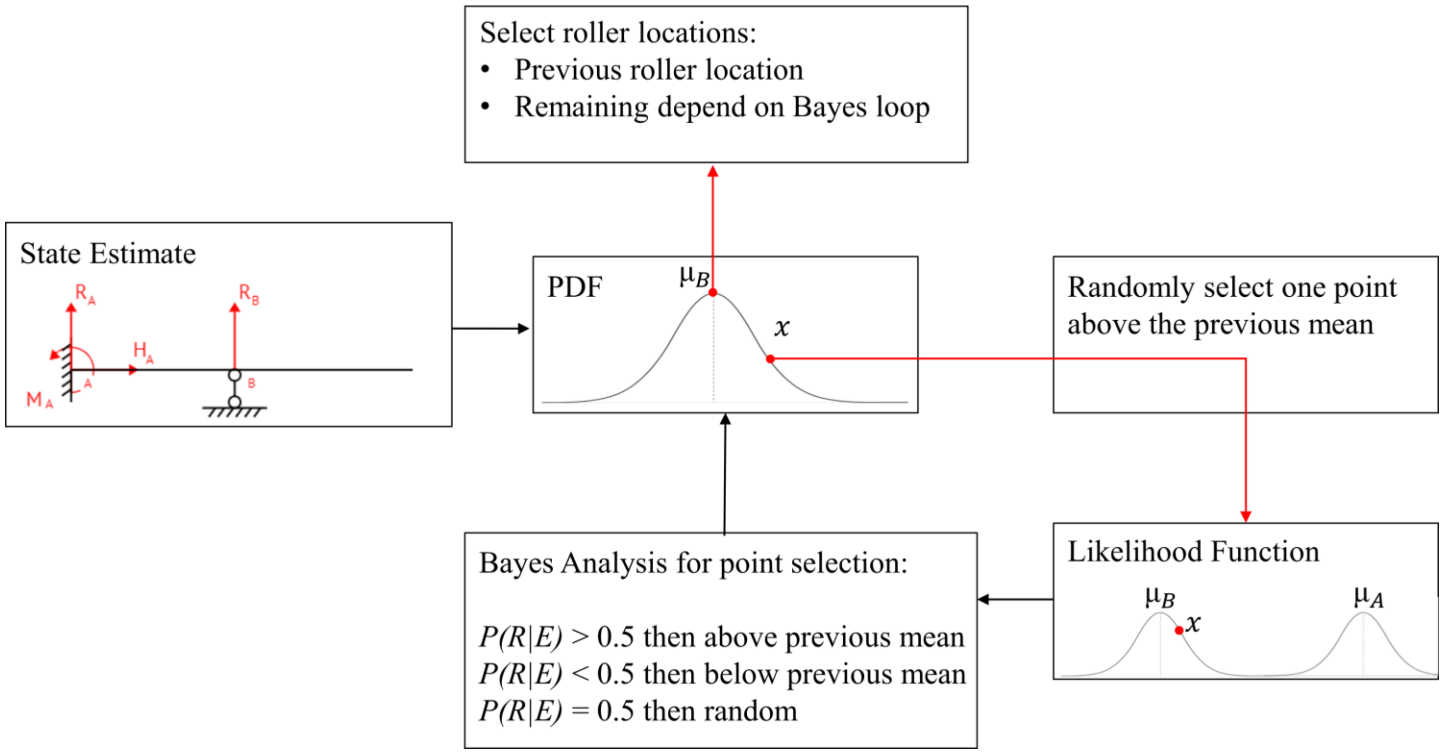
Sampling

Four sampling methods are used for selecting an appropriate roller location on which LEMP is applied for roller location estimation.





Bayesian Inference



$$P(E|R) = \frac{1}{\sqrt{2\pi\sigma^2}} exp \frac{-1}{2} \frac{(x-\mu_B)^2}{2\sigma^2}$$

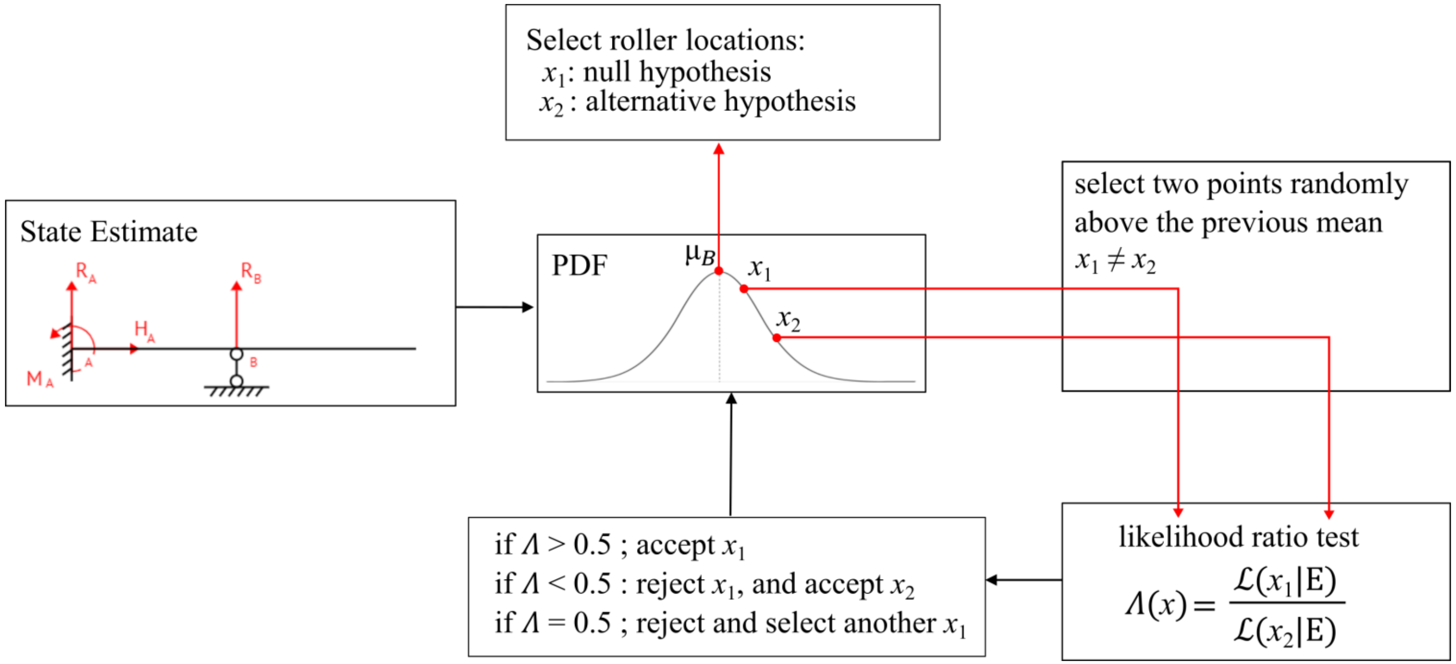
$$P(E|L) = \frac{1}{\sqrt{2\pi\sigma^2}} exp \frac{-1}{2} \frac{(x-\mu_A)^2}{2\sigma^2}$$

σ - Standard deviation

μ - mean

$$P(R|E) = \frac{P(R)P(E|R)}{P(R)P(E|R) + P(L)P(E|L)}$$

Likelihood Ratio Test



Maximum likelihood of model 1

$$\mathcal{L}(x_1|\mathrm{E})$$

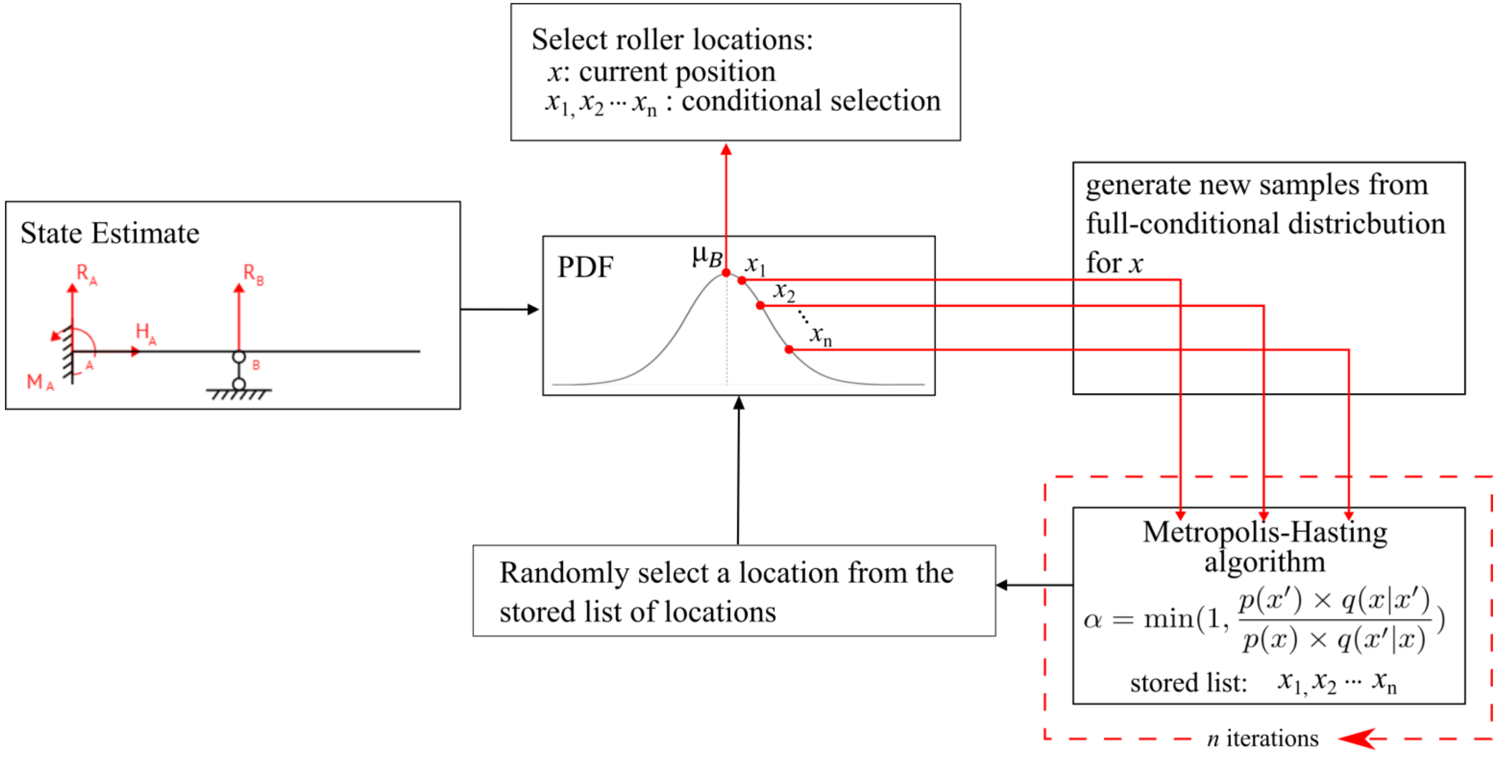
Maximum likelihood of model 2

$$\mathcal{L}(x_2|\mathrm{E})$$

Model acceptability

$$\Lambda$$

Metropolis-Hasting Algorithm

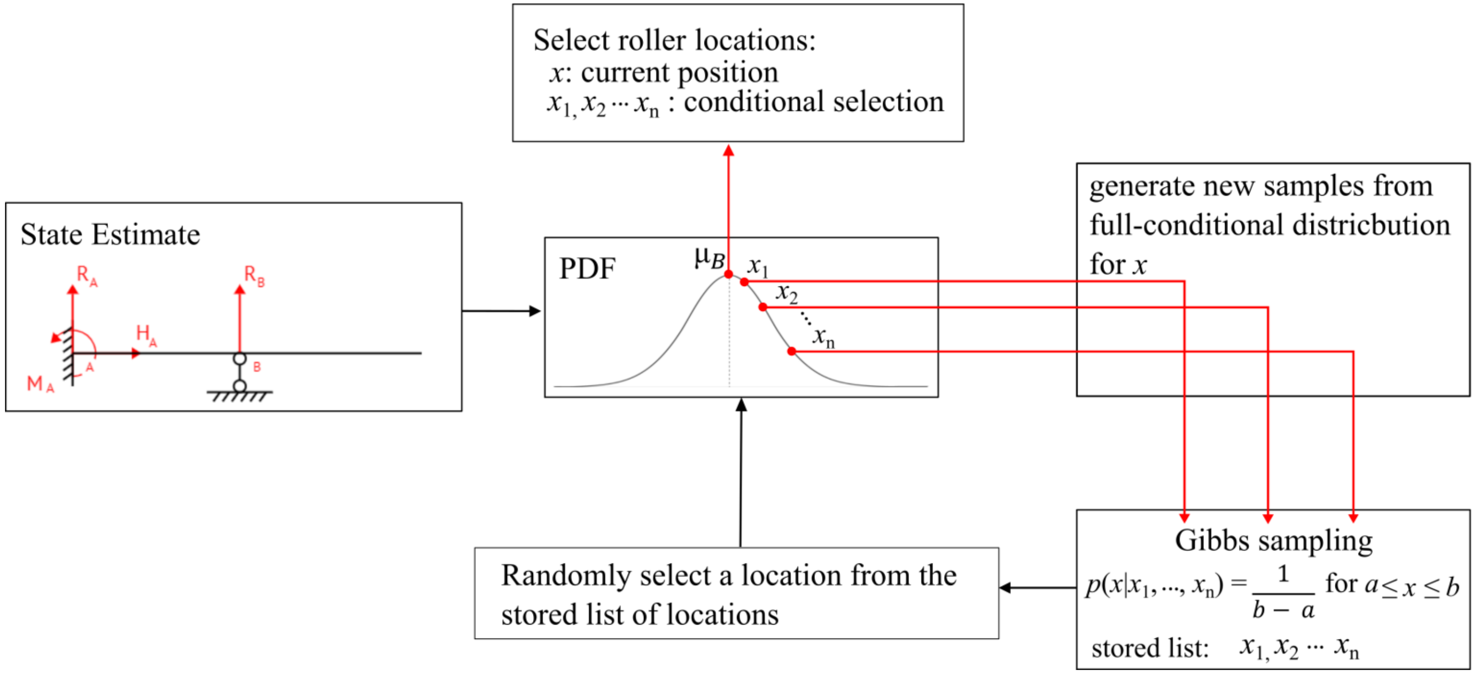


proposal distribution $q(x'|x)$

$$\alpha = \min(1, \frac{p(x') \times q(x|x')}{p(x) \times q(x'|x)})$$

Candidate state is accepted with probability α

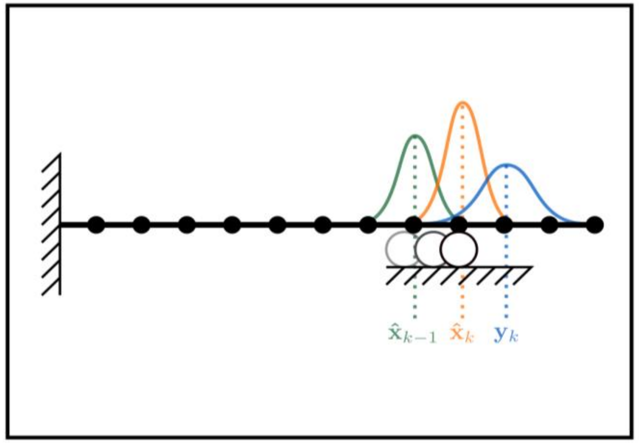
Gibbs Sampling



$p(x)$ the target distribution

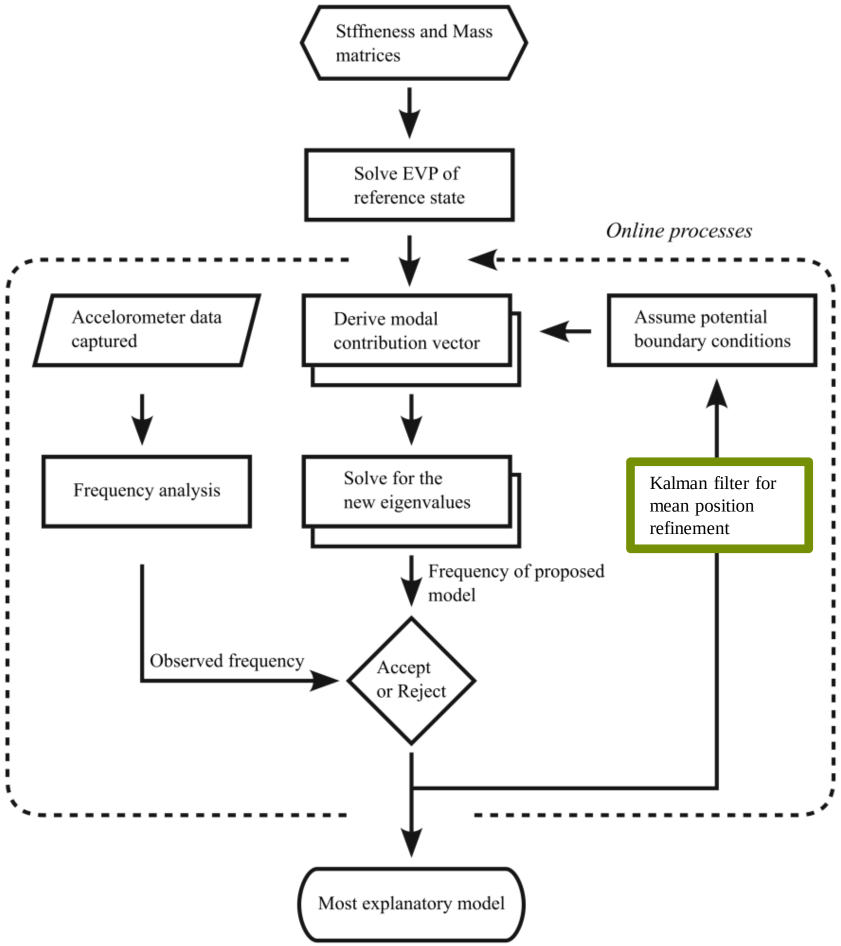
$$p(x|x_1, \ldots, x_n) = \frac{1}{b-a} \text{ for } a \le x \le b$$

Kalman Filter

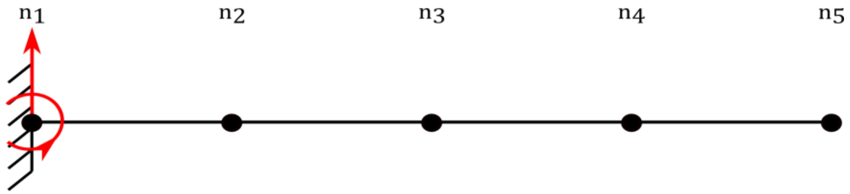


The KF appears to be an ideal solution for isolating and refining the estimation of the roller's true position.

- Provides better estimations for initial sampling guess.
- Offers several advantages, such as linearity and simplicity in implementation
- Adds an extra sequential step that may affect timeliness.



Beam Modeling



Discrete Constant Velocity Model:

$$x_k = Ax_{k-1} + \Omega_p$$
$$y_k = Cx_k + \Omega_m$$

Where we assume that between the $(k-1)$ and k timestep, uncontrolled forces cause a constant velocity

$$x = \begin{bmatrix} p \\ v \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

State equation modeling

state space representation

$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{w}_p$$

$$\mathbf{y}_k = \mathbf{C}_k \mathbf{x}_k + \mathbf{w}_r$$

$\mathbf{x}_k$ - the expected transition

$\mathbf{A}_k$ - transition matrix

$\mathbf{y}_k$ - the measured noisy variable

- Linear relation between

  $\mathbf{C}_k$ - measurement transition matrix

  $\mathbf{y}_k$ - the output from state $\mathbf{x}_k$

- Modeled as a linear Gaussian process

  $\mathbf{w}_r$,$\mathbf{w}_p$ - noise is additive,

  - independently and identically distributed.

a prior estimate

A prior estimate

$$\hat{\mathbf{x}}_{a,k} = \mathbf{A}_k \hat{\mathbf{x}}_{s,k-1}$$

$$\hat{\mathbf{P}}_{a,k} = \mathbf{A}_k \hat{\mathbf{P}}_{s,k-1} \mathbf{A}_k^T + \mathbf{Q}_k$$

$\hat{\mathbf{P}}$ - estimate of the covariance

$\mathbf{A}_k \hat{\mathbf{P}}_{s,k-1} \mathbf{A}_k^T$ - expected noise propagated

$\mathbf{Q}_k$ - quantifies the estimated state covariance

$\hat{}$ - denotes estimated value.

subscript "*a*" - KF prior estimations

Measurement innovation and update

Measurement innovation and update

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{C}_k \hat{\mathbf{x}}_{a,k}$$

$$\mathbf{S}_k = \mathbf{C}_k \hat{\mathbf{P}}_{a,k} \mathbf{C}_k^T + \mathbf{R}_k$$

$$\epsilon_k = \tilde{\mathbf{y}}_k^T (\mathbf{S}_k) \tilde{\mathbf{y}}_k$$

$$\mathbf{L}_k = \hat{\mathbf{P}}_{a,k} \mathbf{C}_k^T \mathbf{S}_k^{-1}$$

$\tilde{\mathbf{y}}_k$ - innovation

$\mathbf{z}_k$ - The measurement

$\mathbf{C}_k \hat{\mathbf{x}}_{a,k}$ - a priori expected measurement

$\mathbf{S}_k$ - innovation covariance/believability of the innovation

$\mathbf{R}_k$ - noise covariance in the innovation

$\mathbf{C}_k \hat{\mathbf{P}}_{a,k} \mathbf{C}_k^T$ - predicted innovation covariance

$\mathbf{L}_k$ - Kalman gain

ϵ_k - Normalized Innovation Squared (NIS) metric

a posteriori state estimate

A posteriori state estimate

a posteriori estimate | a priori estimate | measurement innovation | Kalman gain

$$\hat{\mathbf{x}}_{s,k} = \hat{\mathbf{x}}_{a,k} + \mathbf{L}_k \tilde{\mathbf{y}}_k$$

$$\hat{\mathbf{P}}_{s,k} = (\mathbf{I} - \mathbf{L}_k \mathbf{C}_k)\hat{\mathbf{P}}_{a,k}$$

Subscript "*s*" - measurement update.

- If NIS value is rejected,

  $\mathbf{L}_k$ = zero,

  ~~$\mathbf{L}_k \tilde{\mathbf{y}}_k$~~

  $\hat{\mathbf{x}}_{a,k}$ = best estimate

- If NIS value is falls within the acceptable interval.

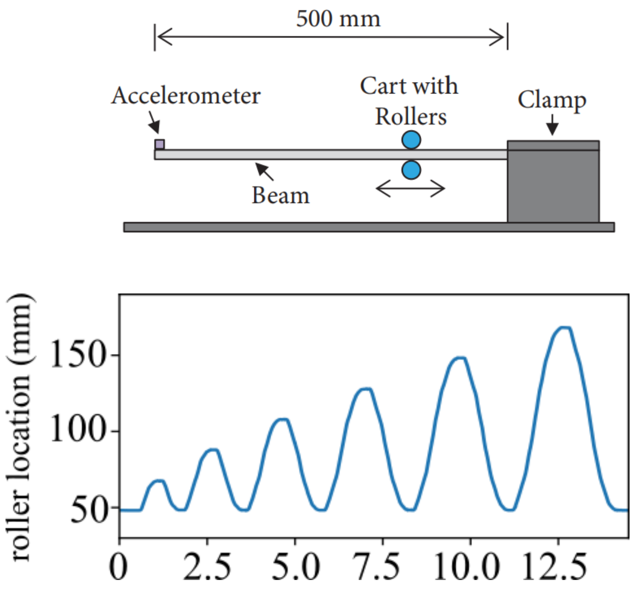
  $\hat{\mathbf{x}}_{s,k}$ - posteriori estimate

  $\hat{\mathbf{P}}_{s,k}$ - covariance

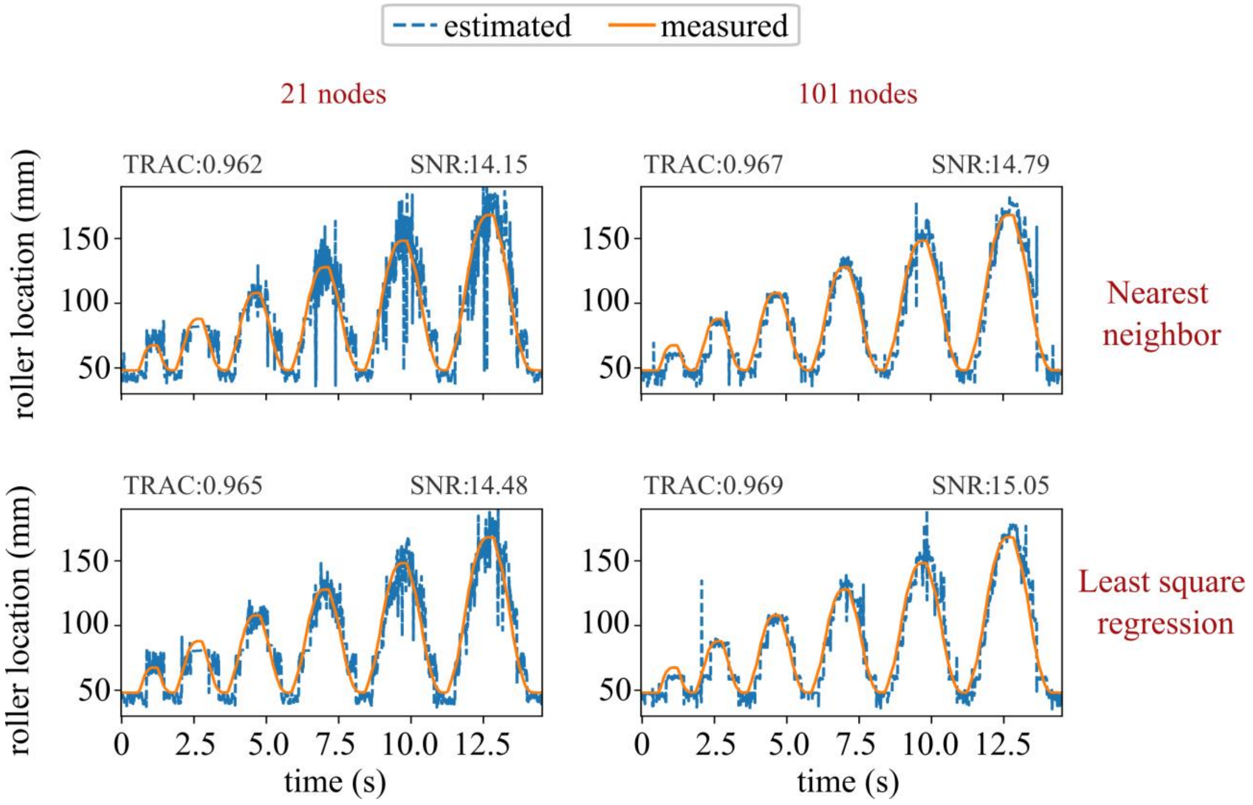
  } Accepted

- **I** - identity matrix.

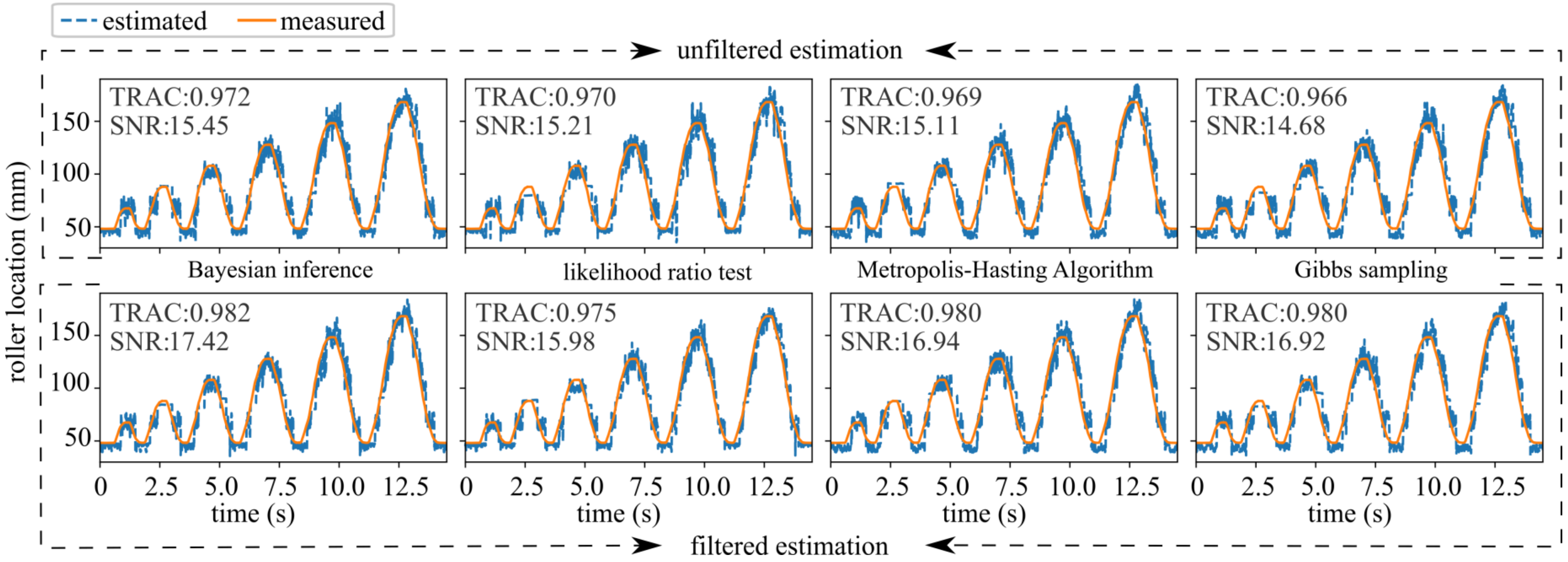
Base state estimation

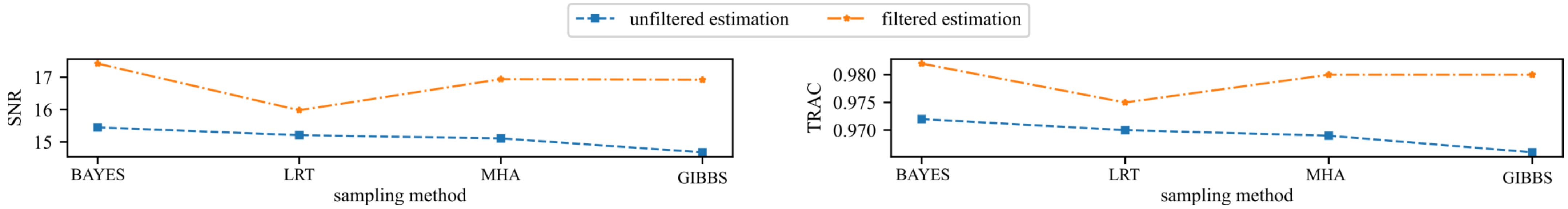


Estimation results obtained using LEMP with a 21-node and 101-node model of the beam and the previously investigated Gaussian sampling technique without the use of a Kalman filter; termed the “base state”.



21-node model state estimation





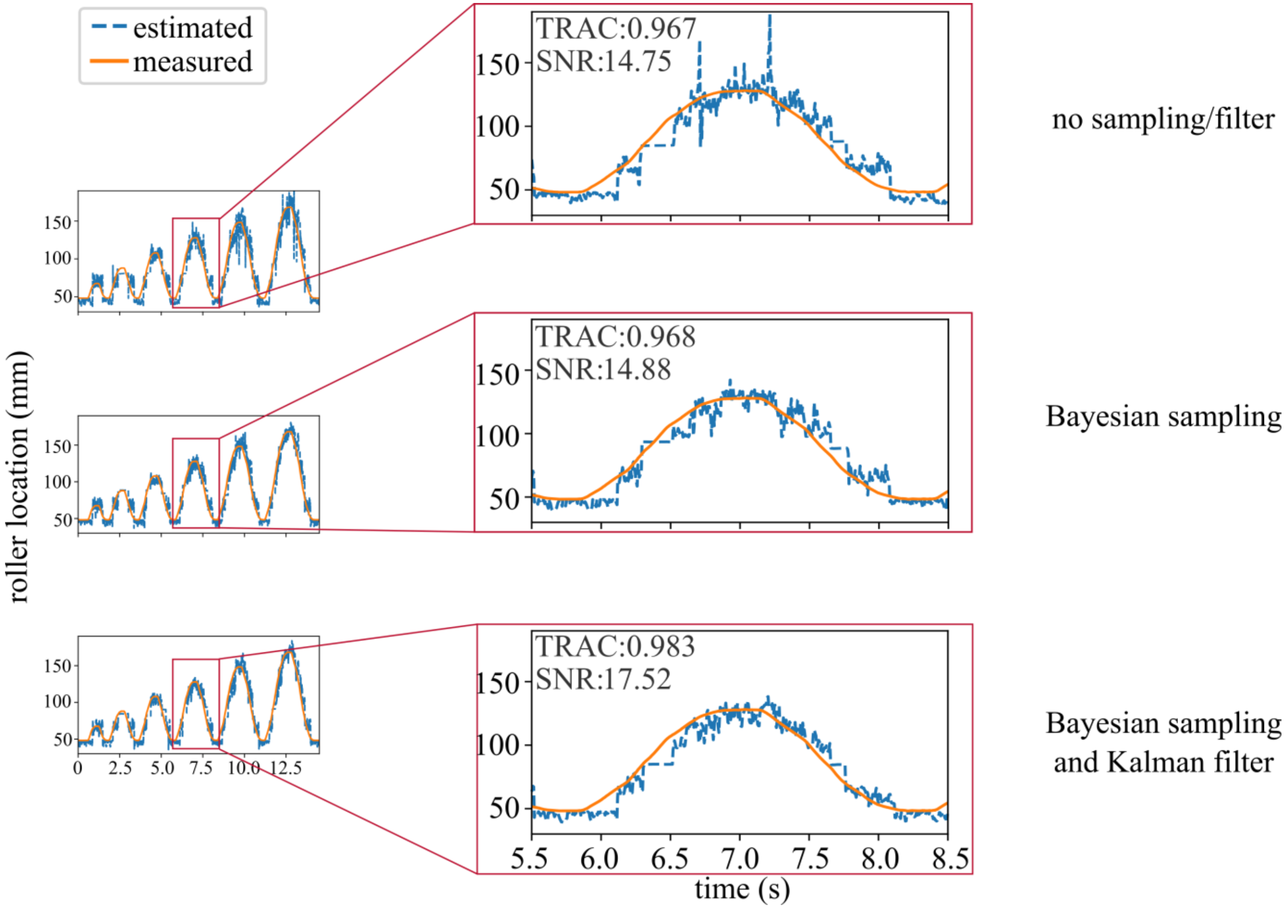
21-node model extended view

Roller position estimation using a 21-node beam model for

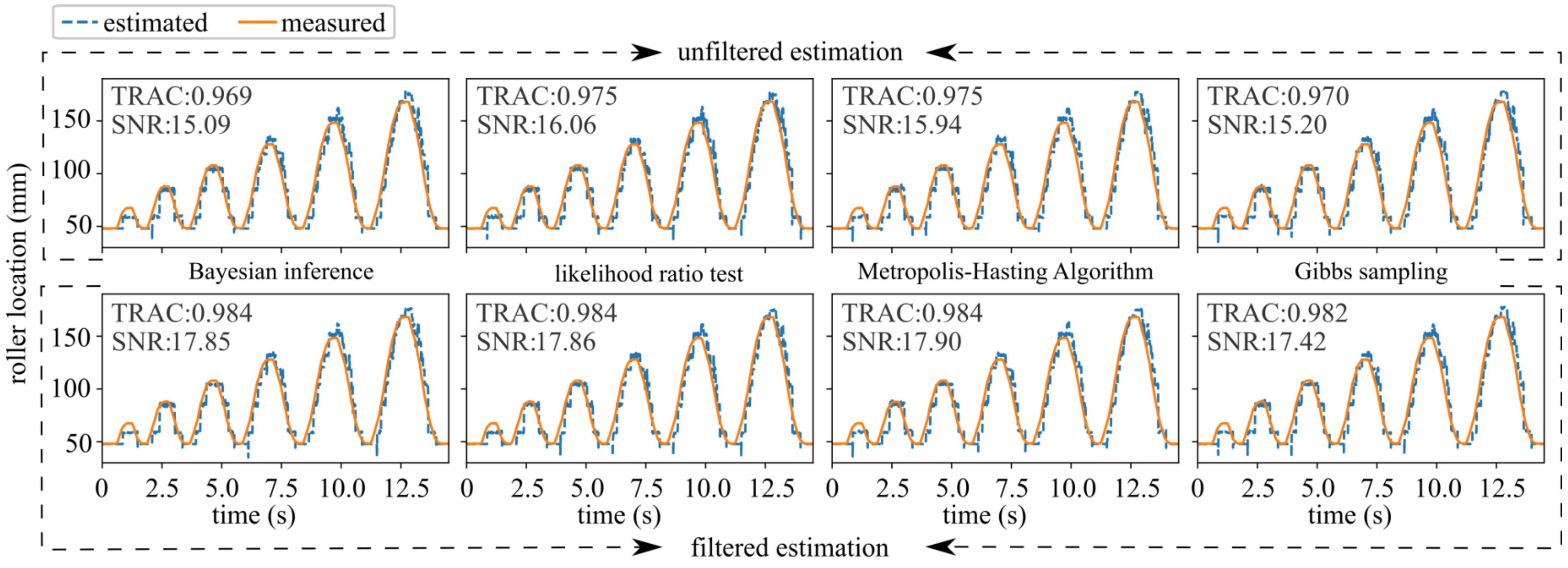
(a) LEMP estimate with no sampling or Kalman filter methodology

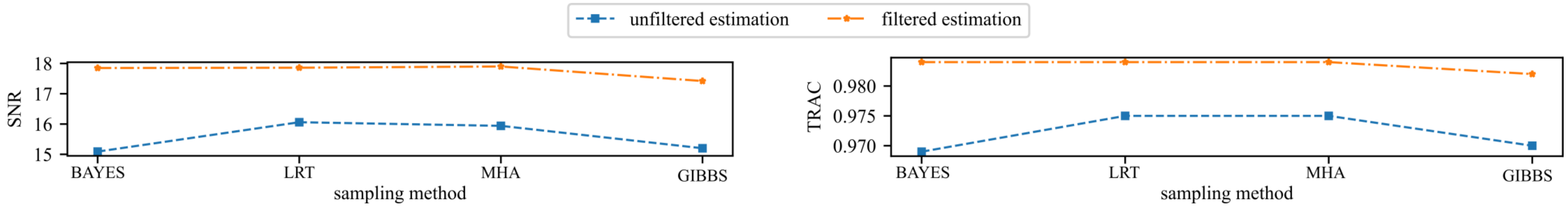
(b) LEMP estimate where roller positions are sampled using Bayesian search space

(c) improved LEMP estimate where roller positions are sampled using the Bayesian search space and also filtered with the Kalman filter .



101-node model state estimation

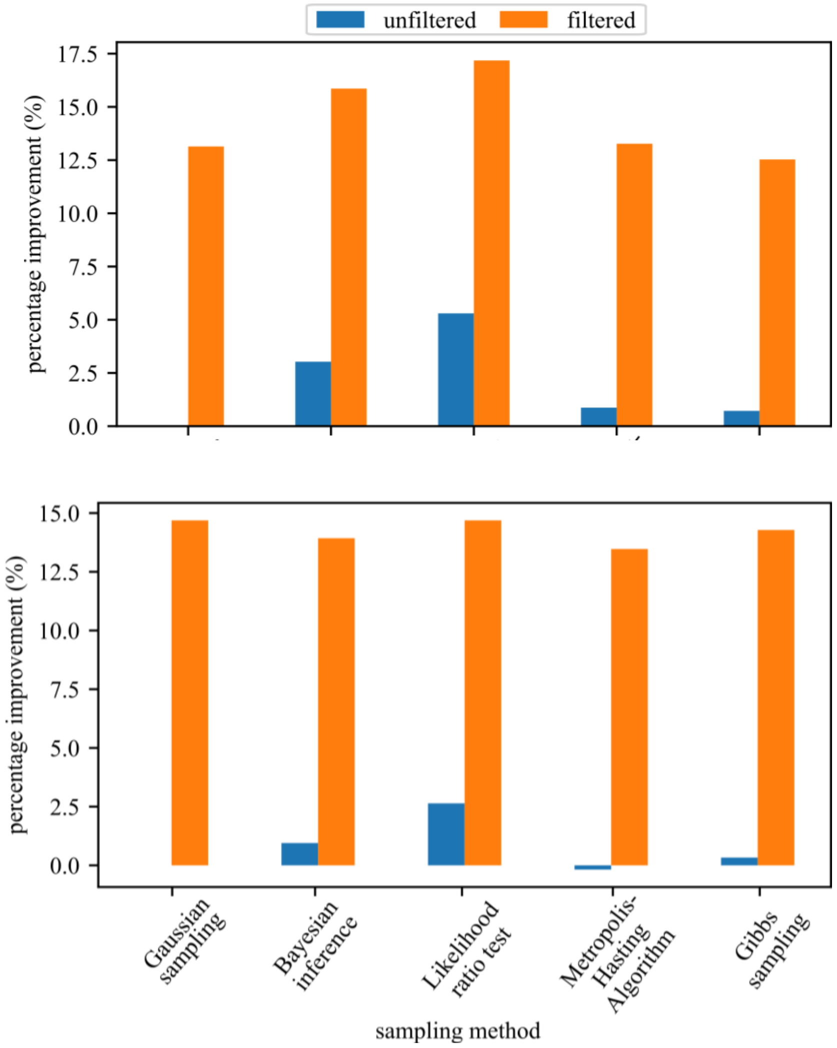




Percentage improvement in estimation

21 nodes

101 nodes



Average percentage improvement in SNR_{dB} compared to estimation without sampling and KF at 21 and 101 nodes for three particle models over 100 trials

	SNR_{dB} improvement			
	21 nodes		101 nodes	
sampling method	unfiltered	filtered	unfiltered	filtered
Bayesian inference	3.03%	15.86%	0.95%	13.93%
likelihood ratio test	5.30%	**17.18%**	2.64%	14.69%
Metropolis-Hasting Algorithm	0.87%	13.27%	-0.18%	13.47%
Gibbs sampling	0.72%	12.53%	0.33%	14.28%
Gaussian sampling	base case	13.14%	base case	14.69%

Conclusion

- The study found that the likelihood ratio test alongside the linear Kalman filter effectively produced accurate results, with an ~17% increase in accuracy for a 21-node model of the considered structure.
- The study also highlighted the importance of filtering outliers, as demonstrated by using the Normalized Innovation Squared (NIS) metric.
- This study successfully improved accuracy over the previous model updating methods, especially for lightweight models with low node counts on all the methodologies tested.

Future Work

- In future work, the LEMP algorithm will be applied to more complex state estimation.

Acknowledgement

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THANKS!

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