

# FINITE ELEMENT MODELING OF FIXED-FIXED BEAMS UNDER SHOCK EXCITATION WITH ACTIVE CONTROL

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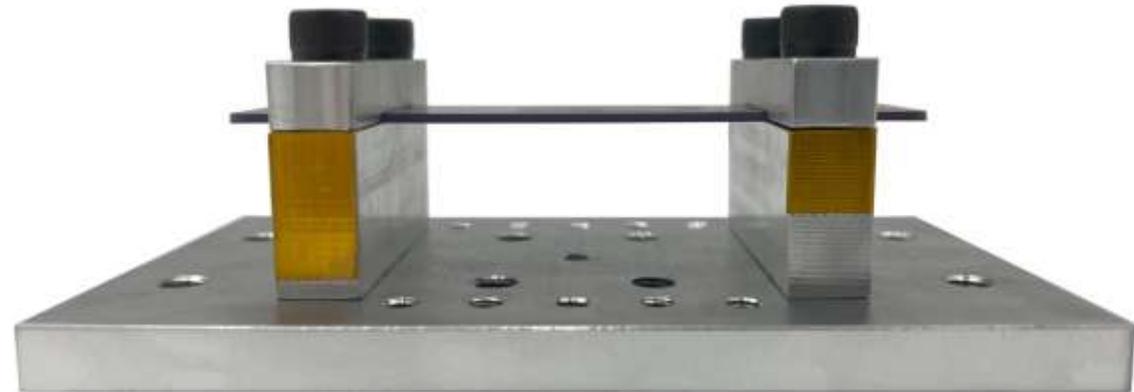


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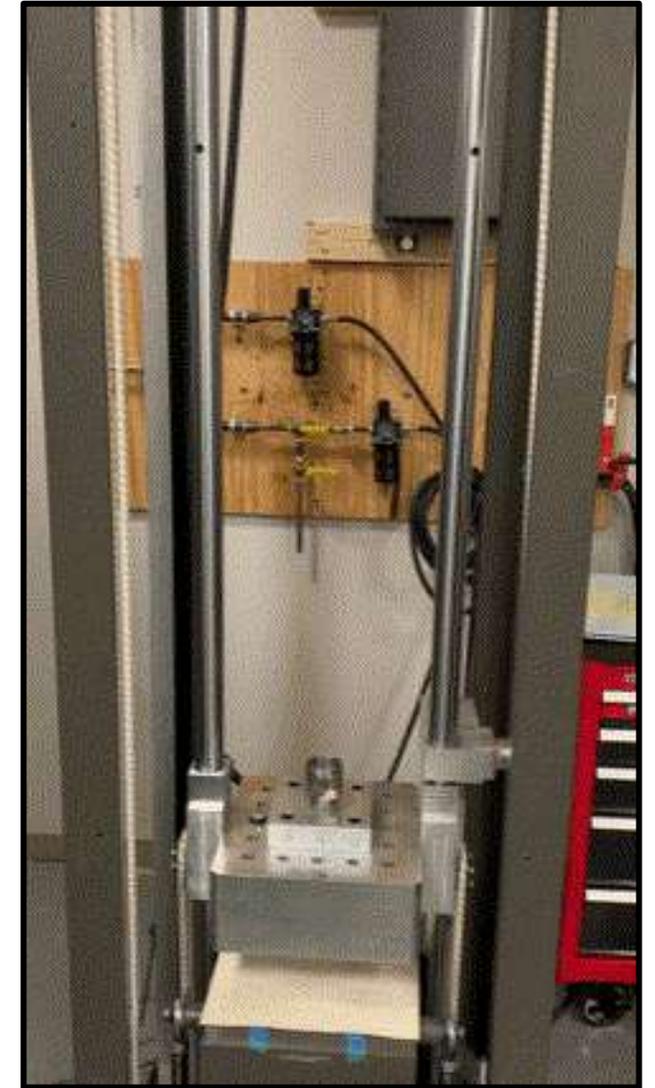
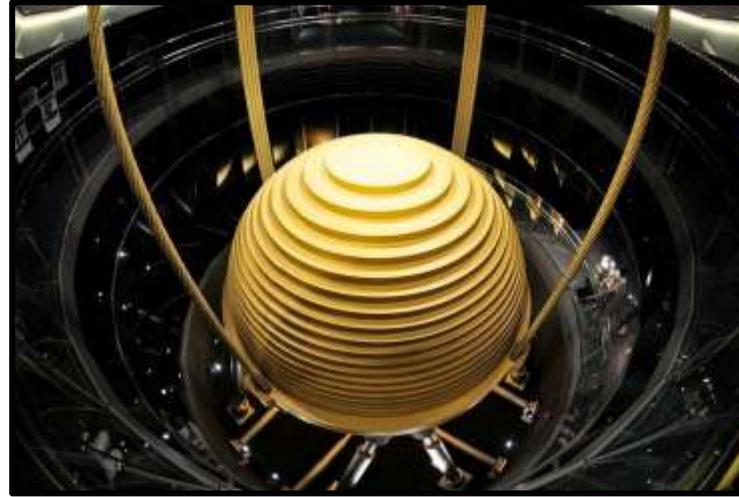
# OVERVIEW

- Introduction
- Ongoing Work
- Project Motivation
- Review on Active Control
- Failure Testing
- Focused Study
  - Simulation Model
  - Parameters
  - Control Implementation
  - Static Analysis
  - Dynamic Analysis
  - Conclusion
- Extended Work



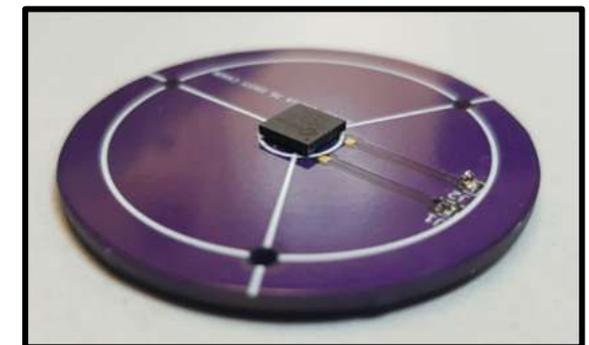
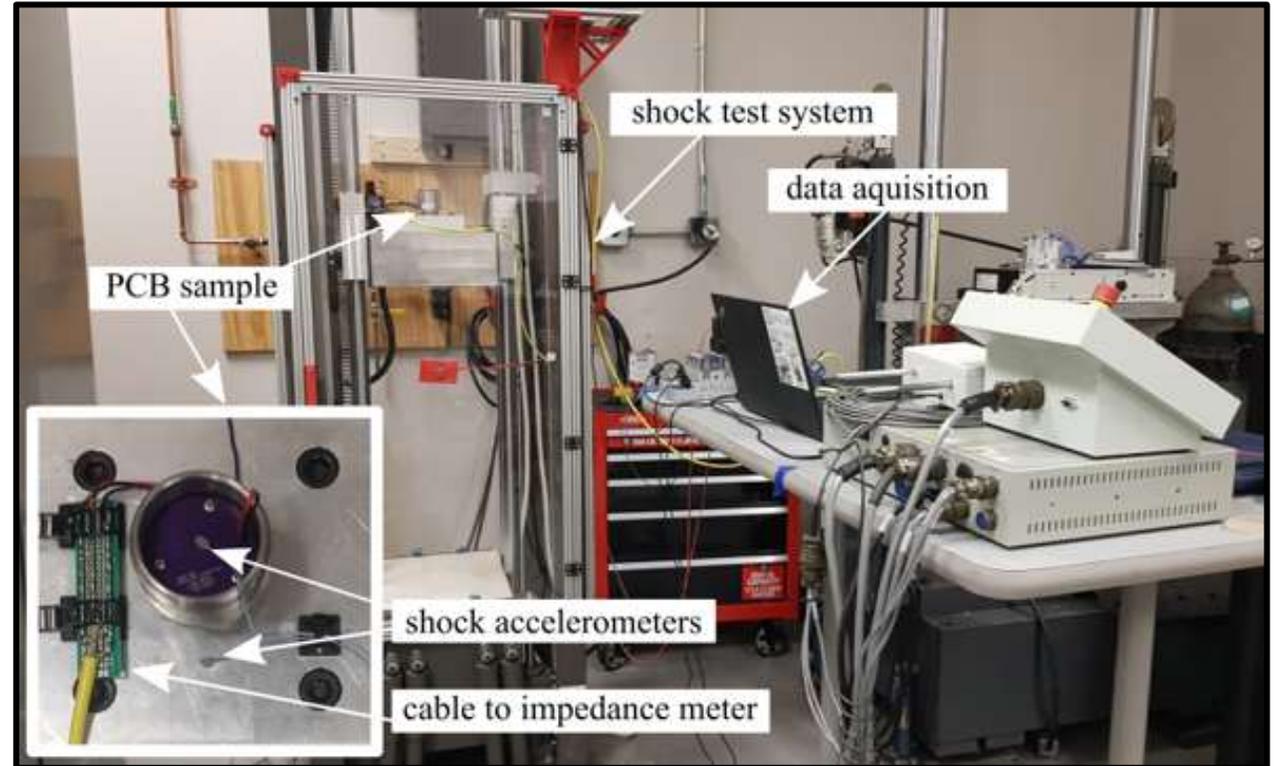
# INTRODUCTION

- Mechanical shock occurs when a system undergoes a dramatic and sudden change in acceleration.
- Shock events can cause damage to the system, contributing to objective failure.
- Active control of these systems can dampen shock and prevent damage.
- In the lab, we plan to use piezoelectric pads to provide sensing and actuation.



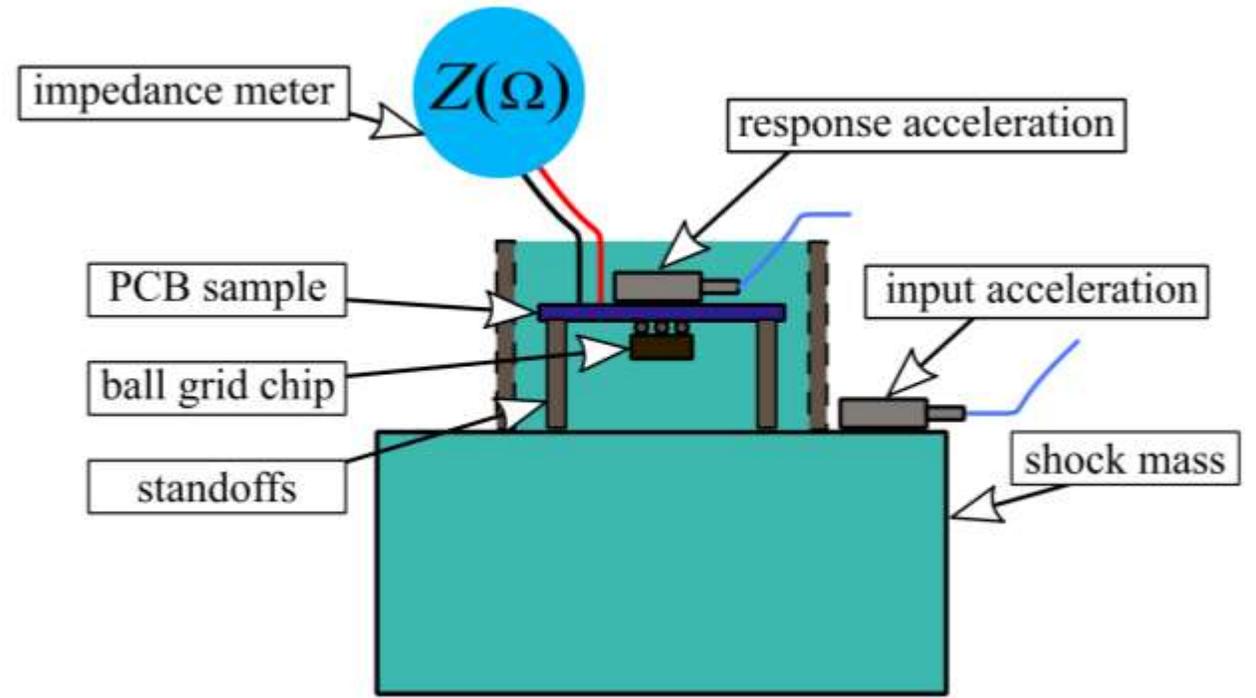
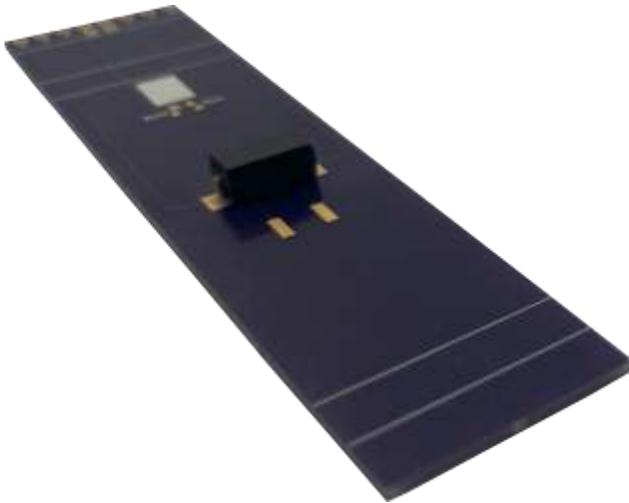
# ONGOING WORK

- Various experiments involving shock testing.
  - Mainly using circular PCB focusing on a USAF application.
  - Other testing using cantilever beams, etc.
- To study PCB and onboard component response to shock.
- Dataset creation for future and related studies.

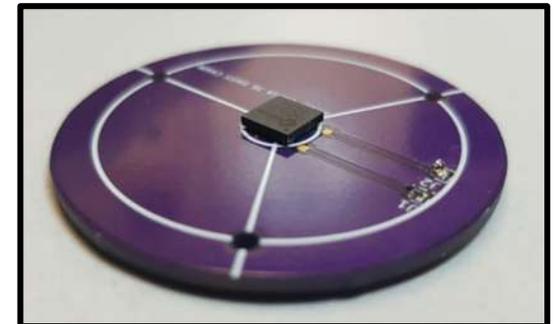
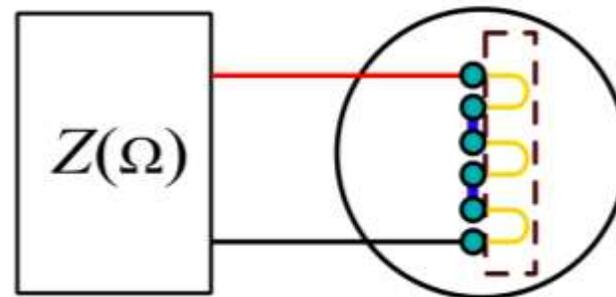


# ONGOING WORK

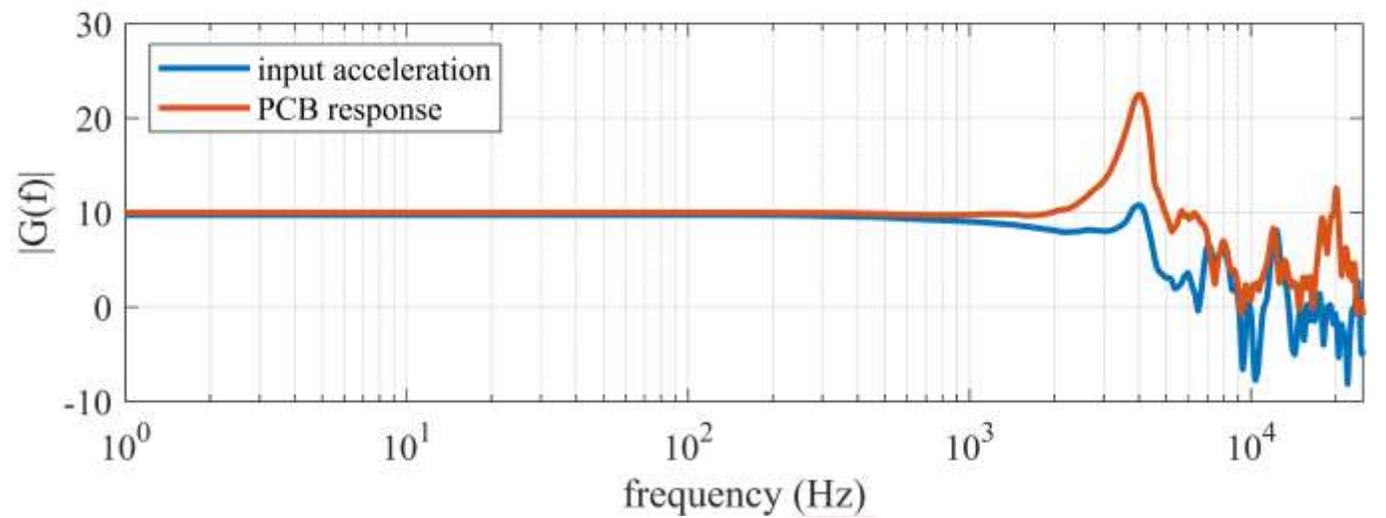
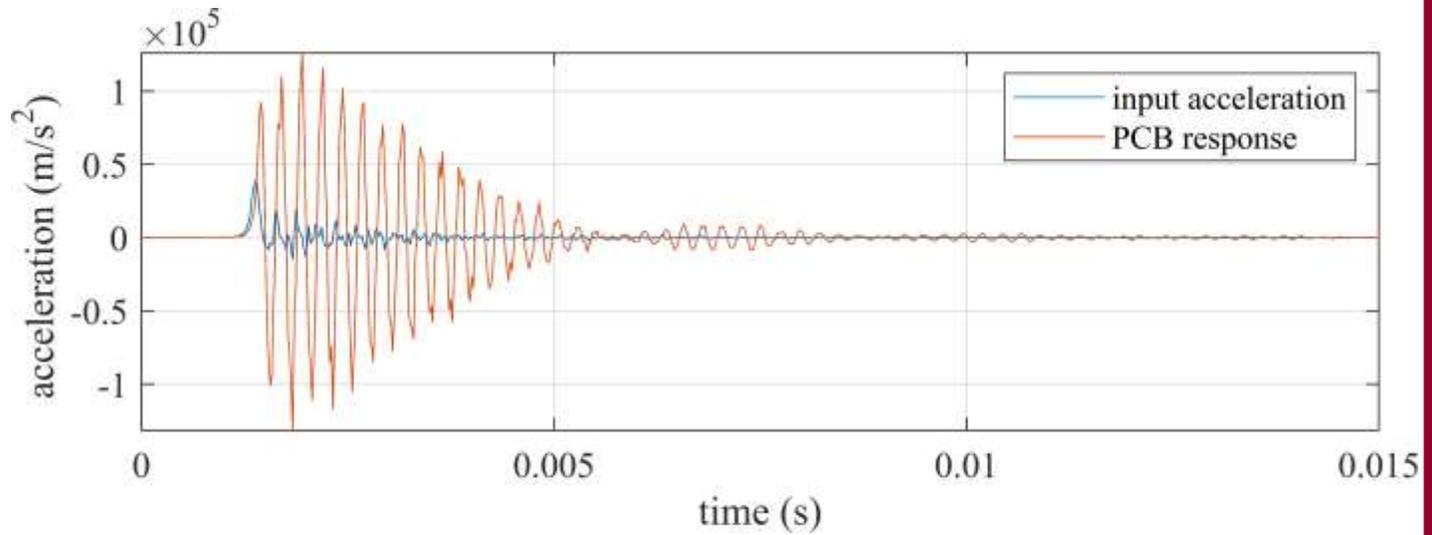
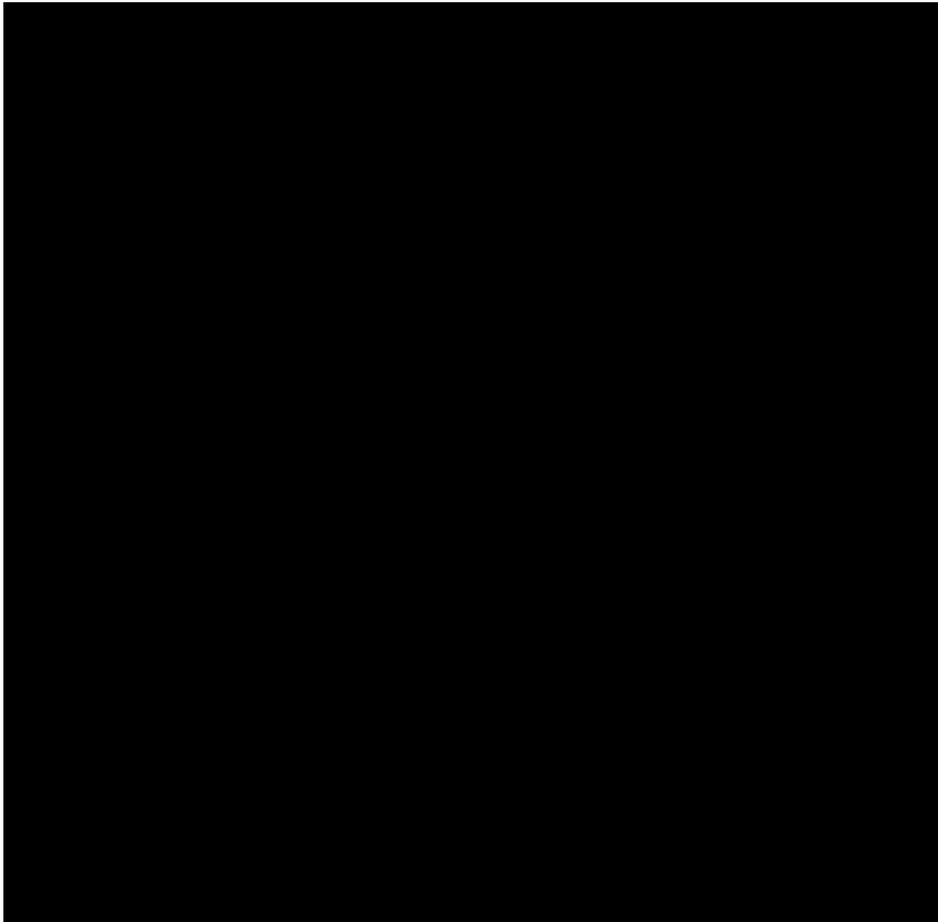
- Printed circuit board design focusing on studying component fatigue rate.
  - Using many different metrics, requiring varying designs of PCB and equipment during experiment.
- Onboard dummy component (BGA) in place of controller.



— PCB connection  
— internal connections



# ONGOING WORK



# REAL-WORLD APPLICATIONS



**Blast Against Civil Structures**

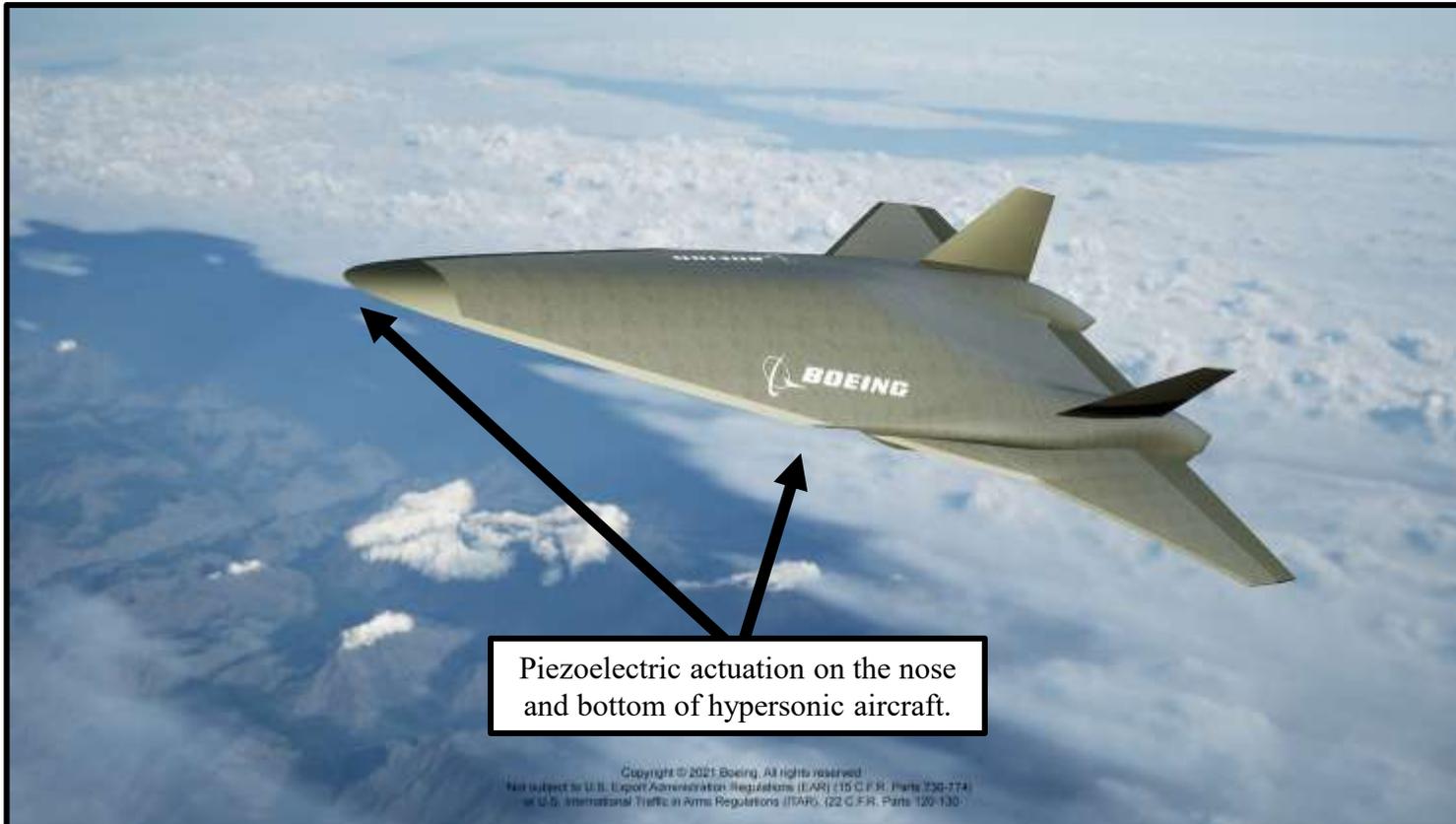


**Automotive Impact and Crashes**



**High-speed Aircraft and Airframes**

# REAL-WORLD APPLICATIONS

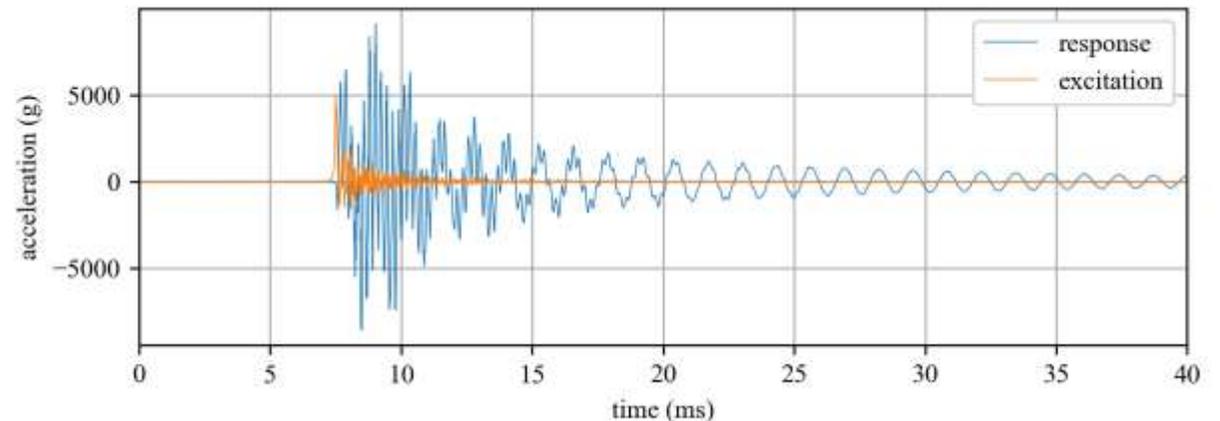
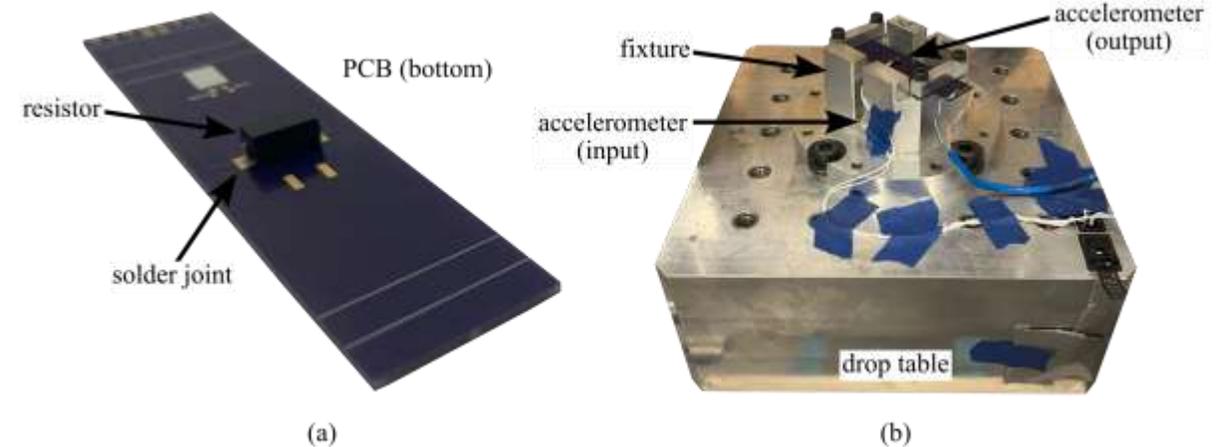


PZT Actuator Patches

# **FAILURE TESTING**

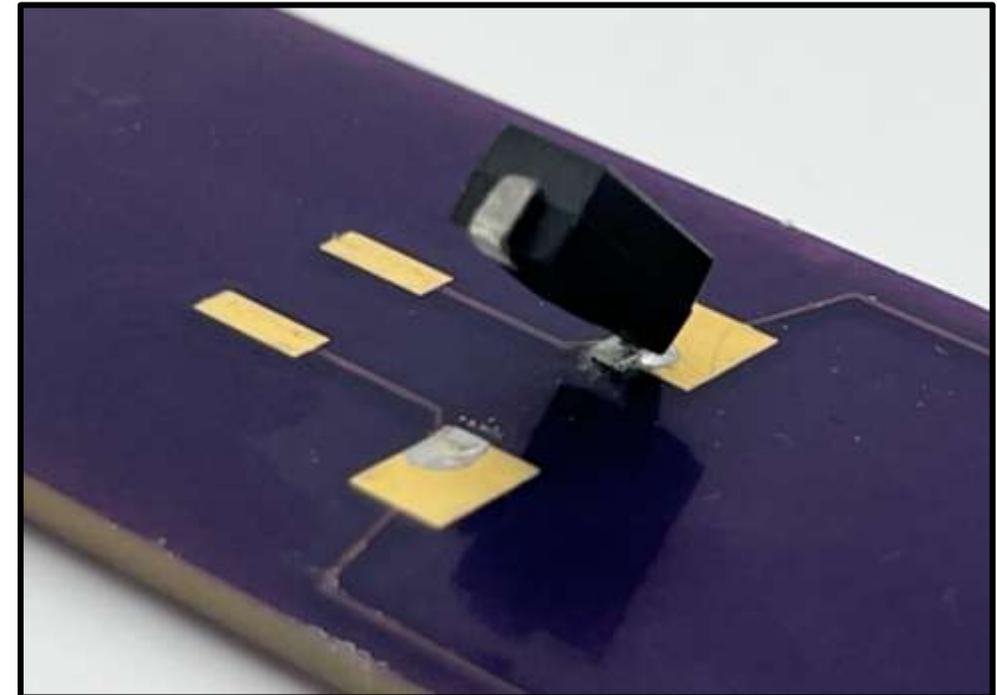
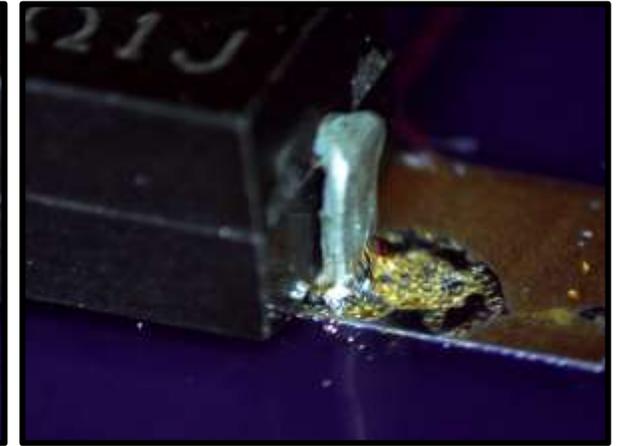
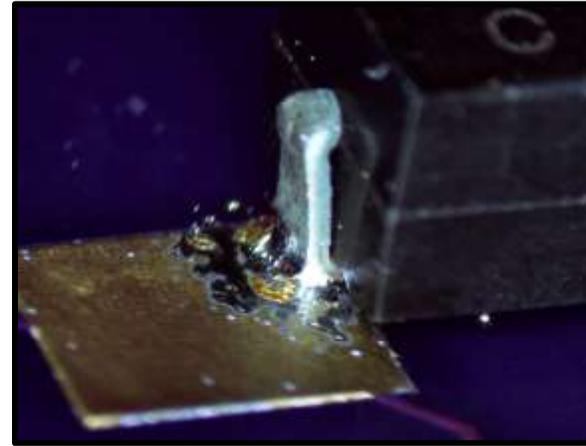
# TEST-TO-FAILURE SETUP

- Printed circuit boards mounted as fixed–fixed beams with surface-mounted resistors.
- Subjected to repeated shock loading at constant impact energy.
- Acceleration measured at the fixture and board midspan to capture input–response behavior.
- Each impact produced  $\approx 5000$  g peak acceleration and sub-millisecond pulse duration.



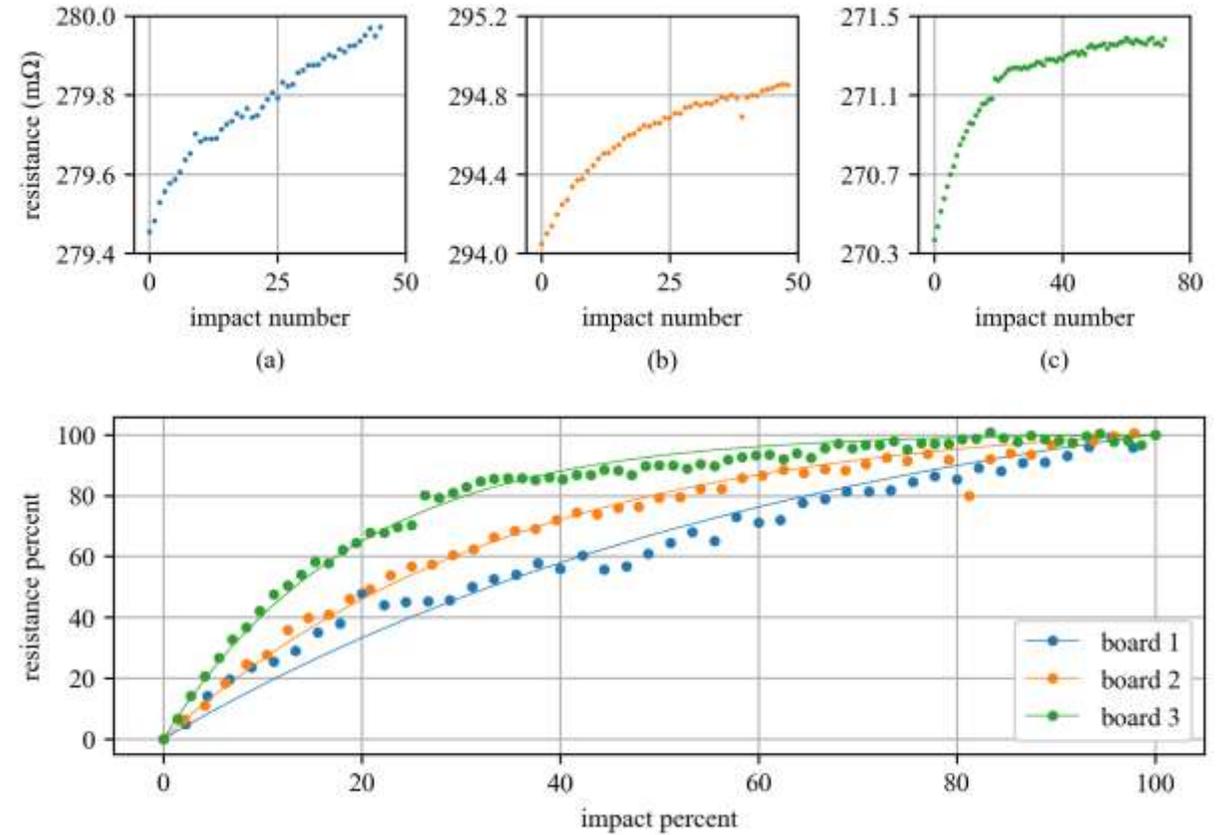
# SYSTEM FAILURE

- Repeated high-g impacts causes crack initiation and growth in solder joints connecting surface-mount resistors.
- Resistor eventually lifted or detached as solder joints fractured.
- PCB substrate remained intact, failure isolated to solder fatigue.
- Consistent failure mode observed across all tested boards.



# DEGRADATION TRACKING

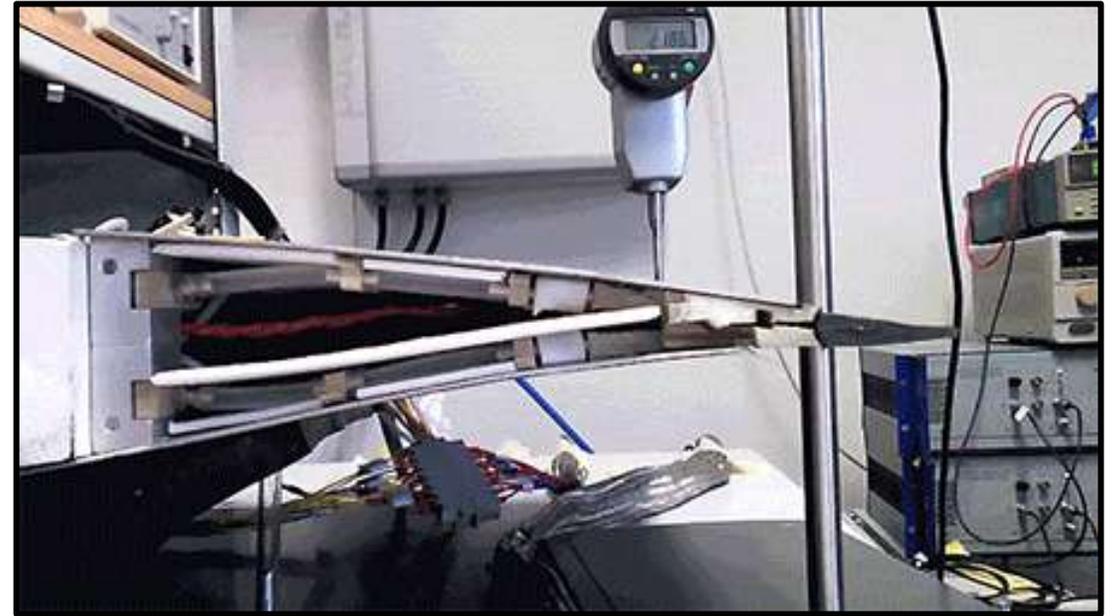
- Electrical resistance monitored after each impact to track solder joint and trace degradation.
- Resistance trends revealed gradual increases until near failure, corresponding to crack initiation in solder joints.
- Demonstrates a repeatable link between mechanical response and electrical degradation in high-rate environments.



**WHY ACTIVE CONTROL?**

# IMPORTANCE OF ACTIVE CONTROL

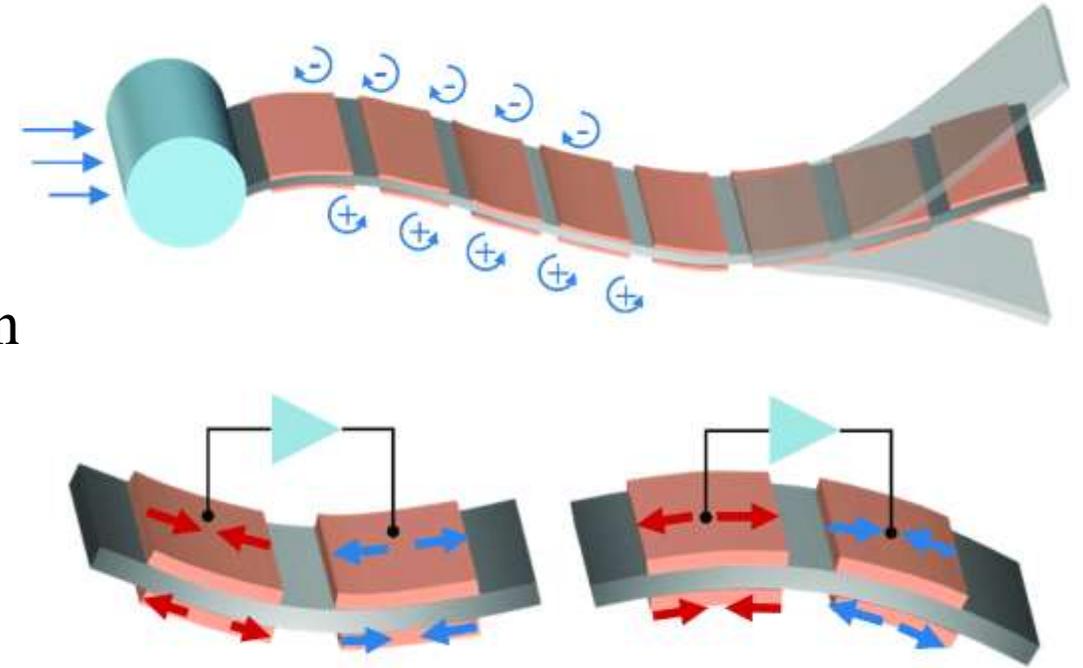
- Passive damping techniques cannot respond fast enough to high-rate, short-duration shock events.
- Active control introduces adaptability and precision, modifying system response in real time.
- Enables control of vibration amplitude, energy dissipation, and structural strain during impact.
- Forms the foundation for smart structures capable of self-adjusting to harsh dynamic environments.



<https://insights.globalspec.com/article/7578/morphing-wing-created-using-smart-materials-and-actuators>.  
(Credit: Institut de Mecanique des Fluides Toulouse)

# PIEZOELECTRIC FUNDAMENTALS

- Piezoelectric materials generate strain under an electric field and produce voltage under deformation.
- When bonded to a structure, they act as integrated sensors and actuators.
- By prestressing the system, the actuator can stabilize the structure after external forces are applied.
- Their high bandwidth, compact form, and bidirectional electromechanical coupling make them ideal for fast control.
- Suitable for lightweight, embedded systems such as printed circuit boards, aerospace panels, and precision equipment.

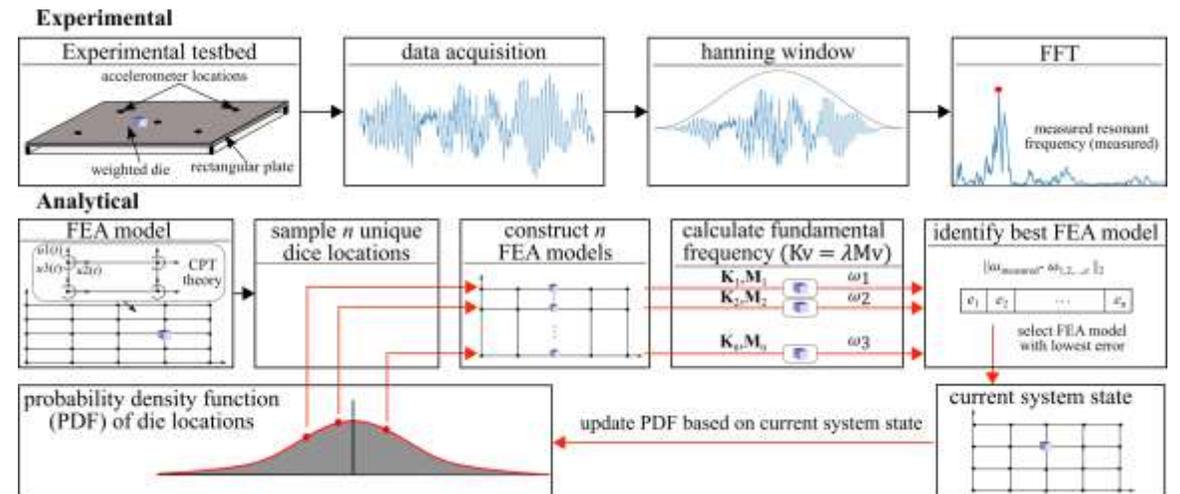


Liu S, Mao J, Liu H, Gao P, Qu Y. Nonlinear flapping and symmetry-breaking bifurcation modulation of a piezoelectric metamaterial beam in viscous flow. *Journal of Fluid Mechanics*. 2025;1019:A2. doi:10.1017/jfm.2025.10556

**NEED FOR SIMPLIFIED MODELS FOR  
EDGE IMPLEMENTATION**

# WHY WE NEED REAL-TIME MODELS

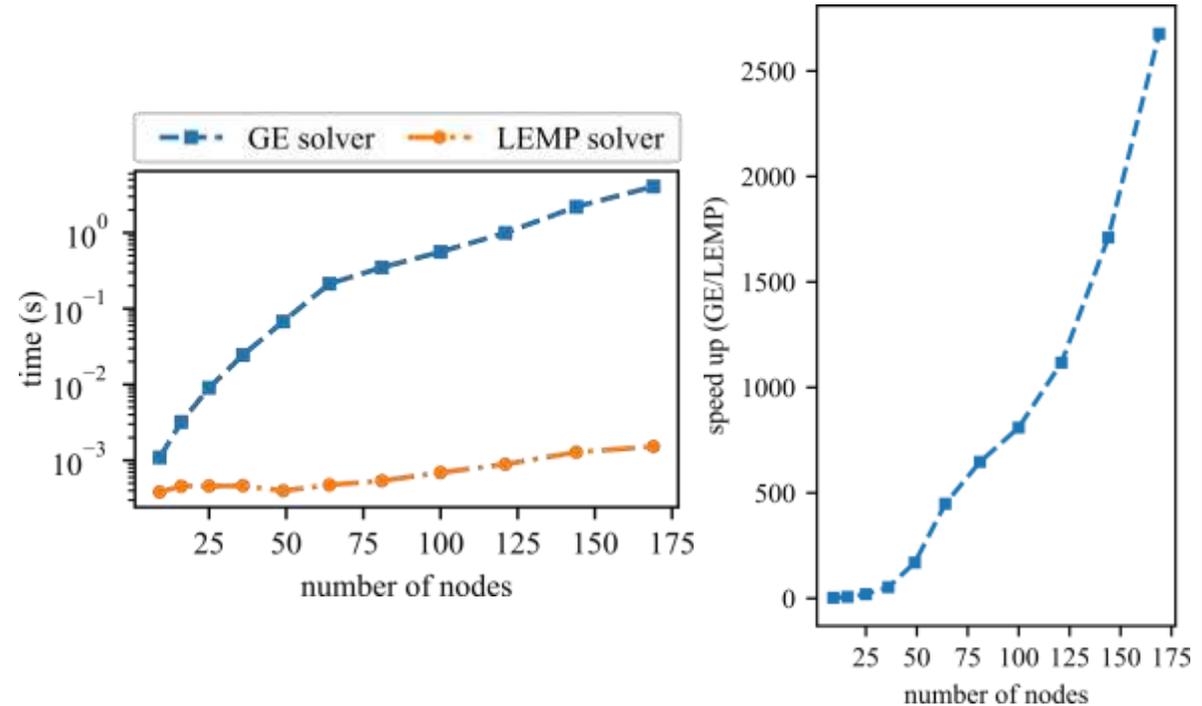
- Conventional FEMs are too large to run during those events
- Need computationally efficient, physics-based models that can update and respond within milliseconds
- Goal: enable edge-level decision making for sensing, control, and damage detection



Emmanuel A. Ogunniyi, Alexander B. Vereen, and Austin R. J. Downey. Microsecond model updating for 2D structural systems using the local eigenvalue modification procedure. In *Proceedings of the 14th International Workshop on Structural Health Monitoring, shm2023*. Destech Publications, Inc., September 2023. doi:10.12783/shm2023/36937

# EXISTING MODEL-UPDATING FRAMEWORKS

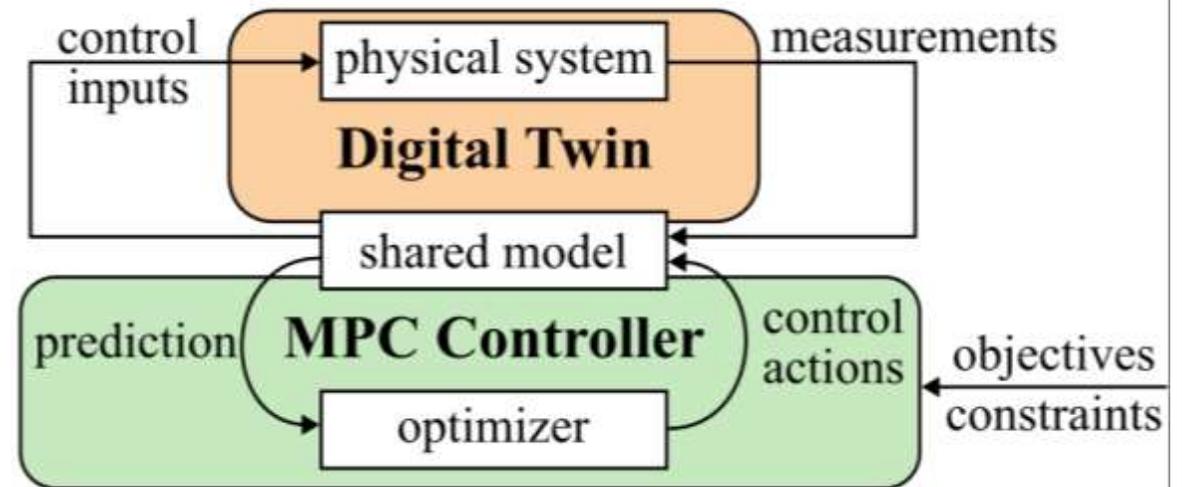
- Prior work showed millisecond model updating using reduced FEMs
- Algorithms such as Local Eigenvalue Modification Procedure (LEMP) can identify stiffness or mass changes in real time
- Bottleneck: full FEM eigenvalue solutions dominate computation time
- Edge-deployment requires models that are compact, 1D/2D, and easily modified



Emmanuel A. Ogunniyi, Alexander B. Vereen, and Austin R. J. Downey. Microsecond model updating for 2D structural systems using the local eigenvalue modification procedure. In *Proceedings of the 14th International Workshop on Structural Health Monitoring, shm2023*. Destech Publications, Inc., September 2023. doi:10.12783/shm2023/36937

# ENHANCED DT-BASED CONTROL

- A digital twin is a live copy of the system, so our model is always up to date.
- With a fresh model, the controller can predict what will happen next more accurately.
- That makes choosing control actions easier and smoother (less overshoot, faster settling).
- We can test control moves in the twin first, then send the safe one to the real system.

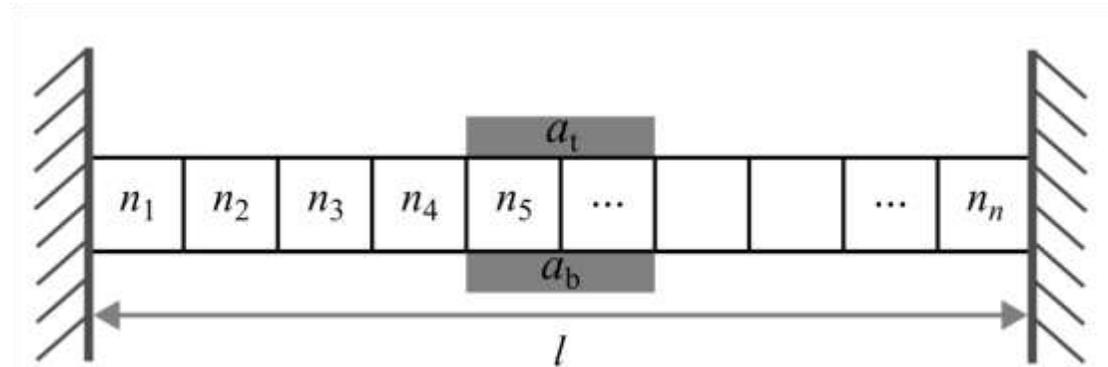
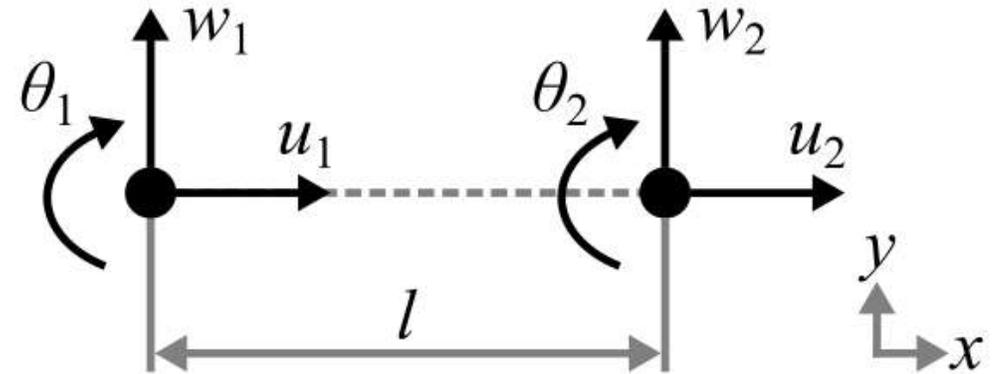


# **SIMULATION AND CONTROL**

**FINITE-ELEMENT MODELING AND ACTIVE VIBRATION SUPPRESSION**

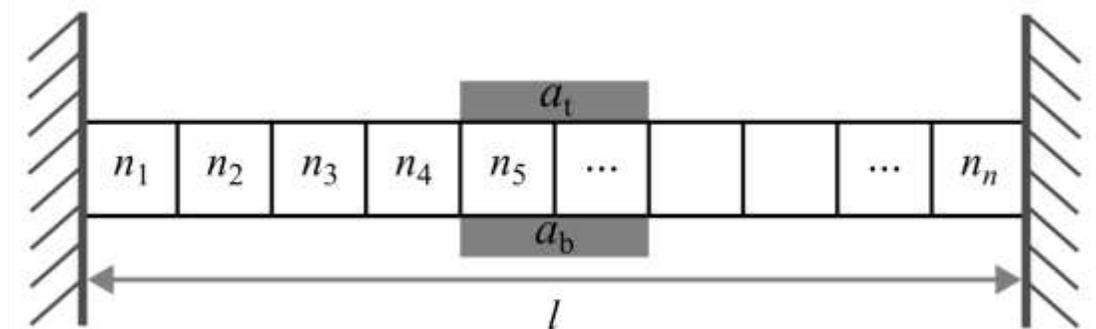
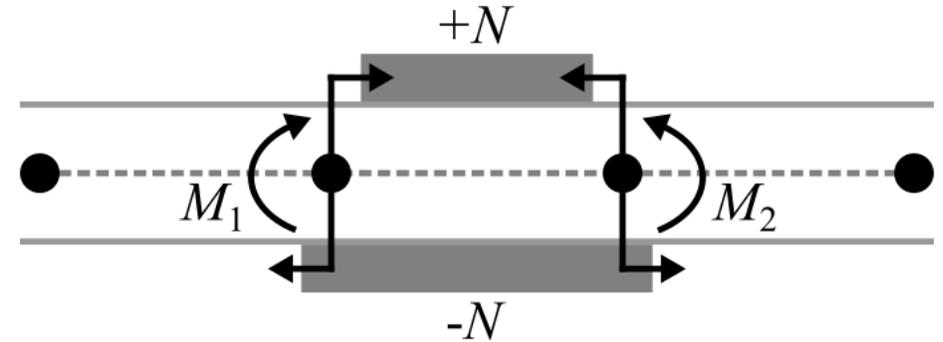
# EXTENDING THE EULER-BERNOULLI BEAM

- Classical E-B beam only models vertical bending, assuming negligible axial coupling.
- Our formulation adds axial DOFs at each node ( $u$ ,  $w$ ,  $\theta$ ).
- Enables simulation of lateral and rotational effects from surface-mounted actuators.
- Captures geometric stiffening and moment-induced curvature within a 1D framework.



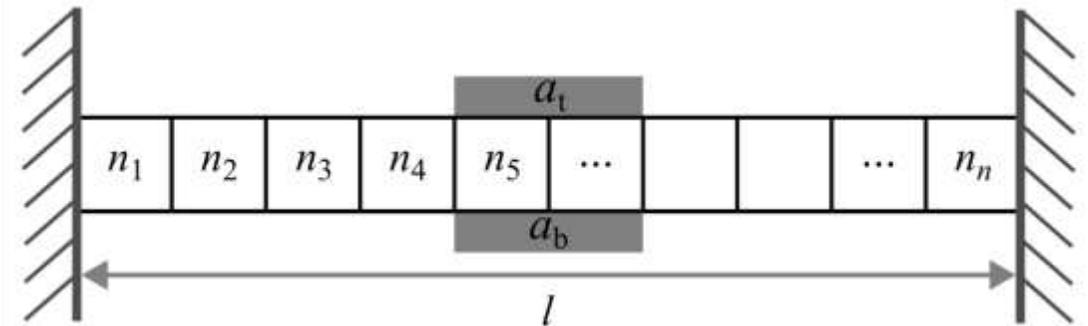
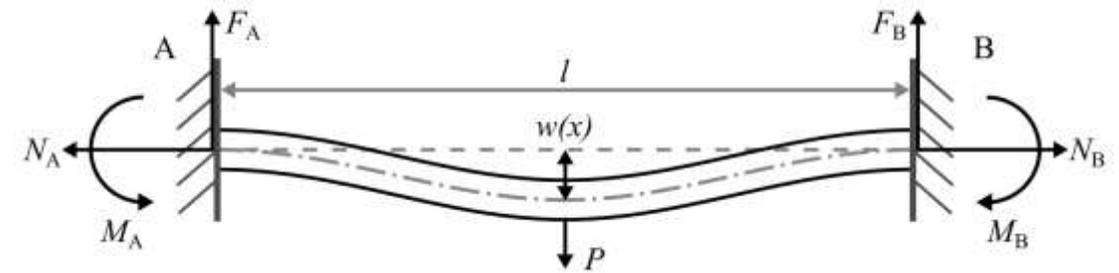
# MOMENT-BASED ACTUATION VIA ROTATIONAL DOFS

- Piezoelectric patches apply equal and opposite torques on top/bottom beam surfaces.
- Modeled as moment couples at adjacent rotational DOFs.
- Equivalent to the bending moment created by real actuators.
- Enables active control without a full 3D electromechanical model.
- Provides localized damping through curvature-feedback PID control.



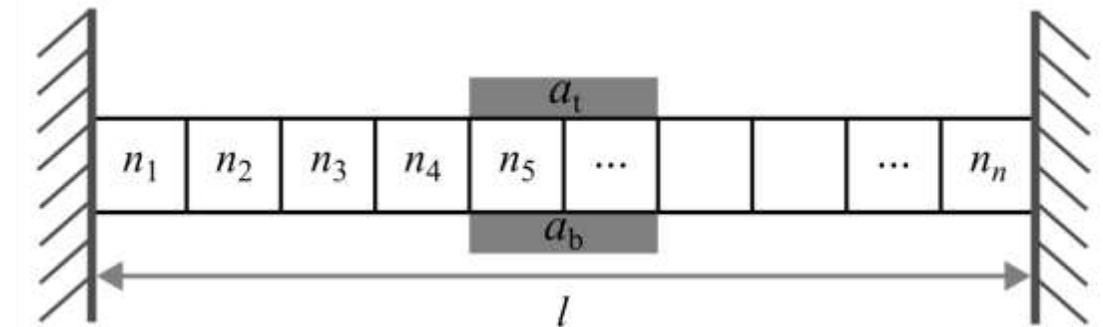
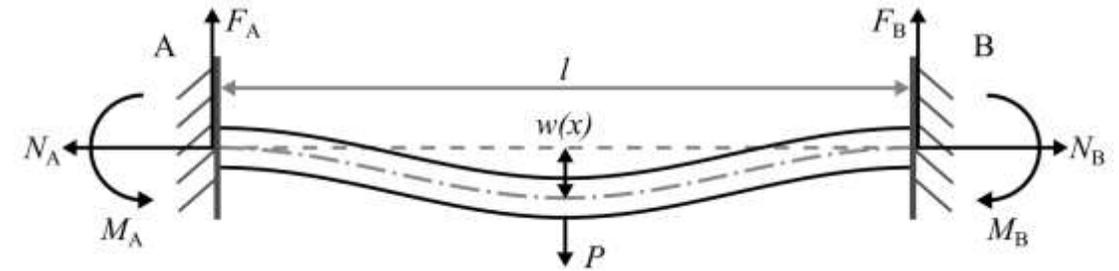
# SIMULATION MODEL OVERVIEW

- 1D Extended Euler–Bernoulli beam with axial, transverse, and rotational DOFs
- Represents a fixed-fixed PCB beam under shock loading ( $\approx 5000$  g, sub-ms pulse)
- Axial and bending coupling included for realistic stiffness behavior
- Rayleigh damping + Newmark-Beta integration used for transient response



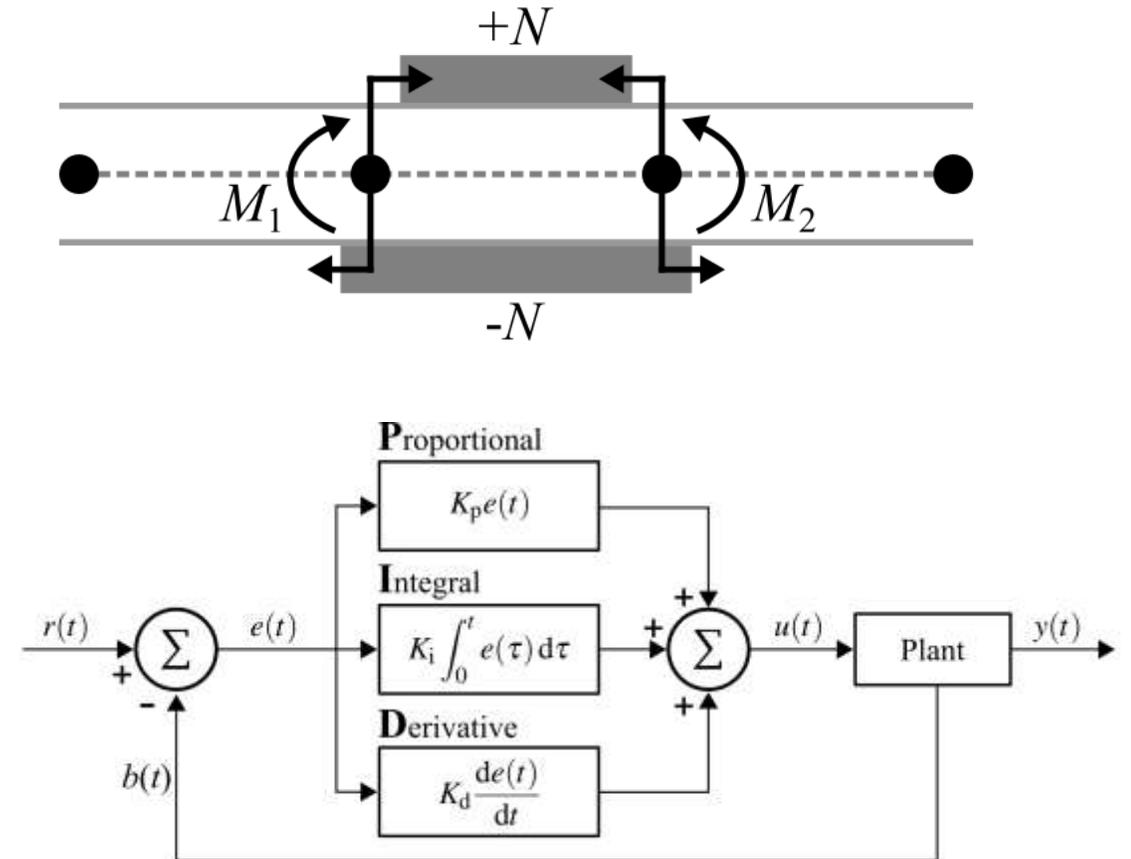
# PARAMETERS

- Material:
  - Based on 175 Tg FR4 PCB beam.
  - Width = 25.4 mm; Thickness = 1.6 mm; Length = 88.9 mm
  - Young's Modulus = 18.6e9 Pa; Density = 1900 kg/m<sup>3</sup>
  - $\alpha = 65.53$ ;  $\beta = 2.95e-6$
- Simulation:
  - Nodes ( $n$ ) = 50; DOFs = 150
  - Impact node =  $\frac{1}{2} n$
  - Control node = (16, ..., 34)  $n$
  - Impact Force = 30 N
  - Impact Duration = 0.1 ms
  - $\beta_n = 0.25$ ;  $\gamma = 0.5$



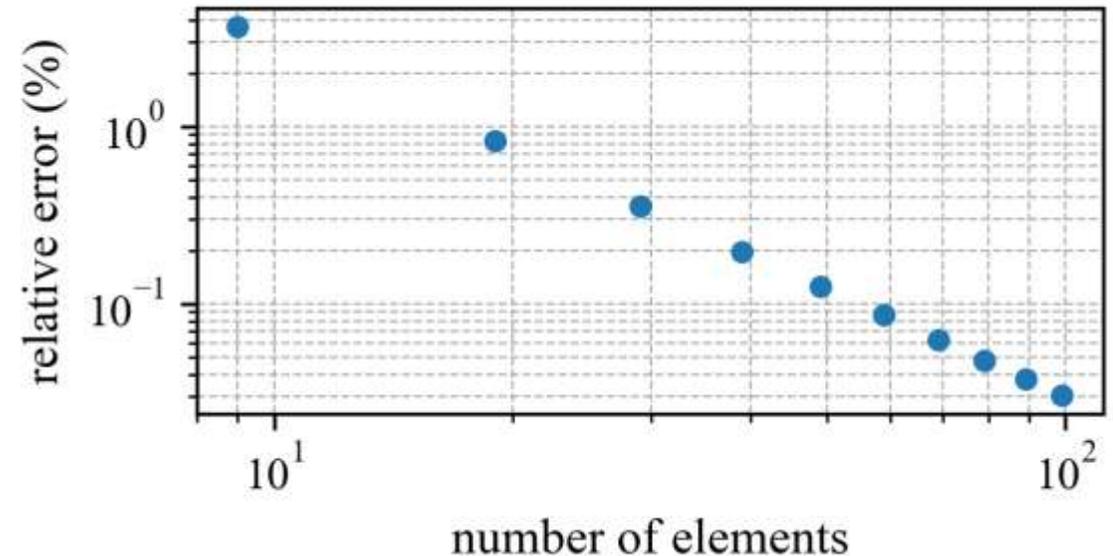
# CONTROL IMPLEMENTATION

- Moment-based actuation mimics piezoelectric patches bonded top/bottom
- Controller applies equal/opposite torques at selected beam nodes
- PID law based on relative rotation (curvature) between nodes
- Control shuts off once response drops below 7.5 % of free peak



# FINITE ELEMENT MODEL CONVERGENCE VALIDATION

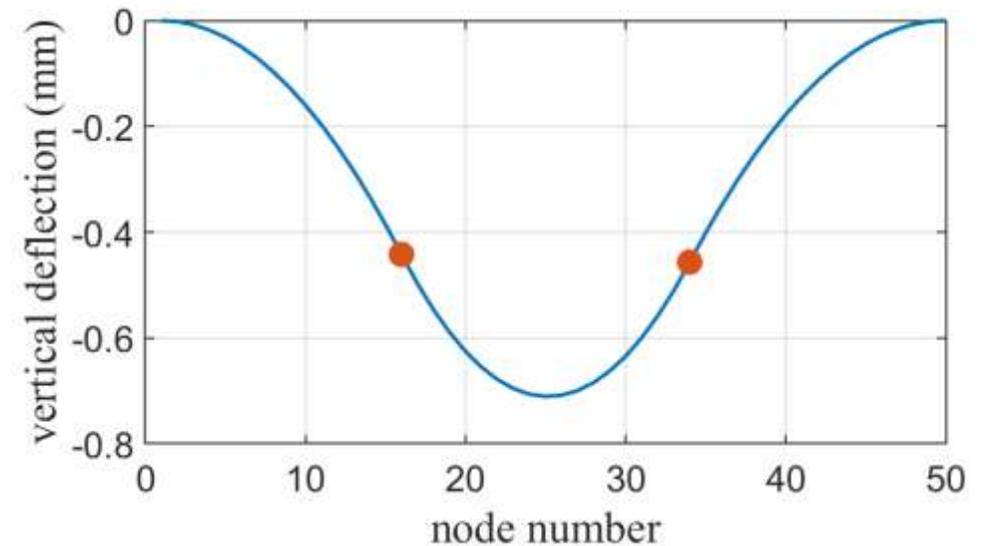
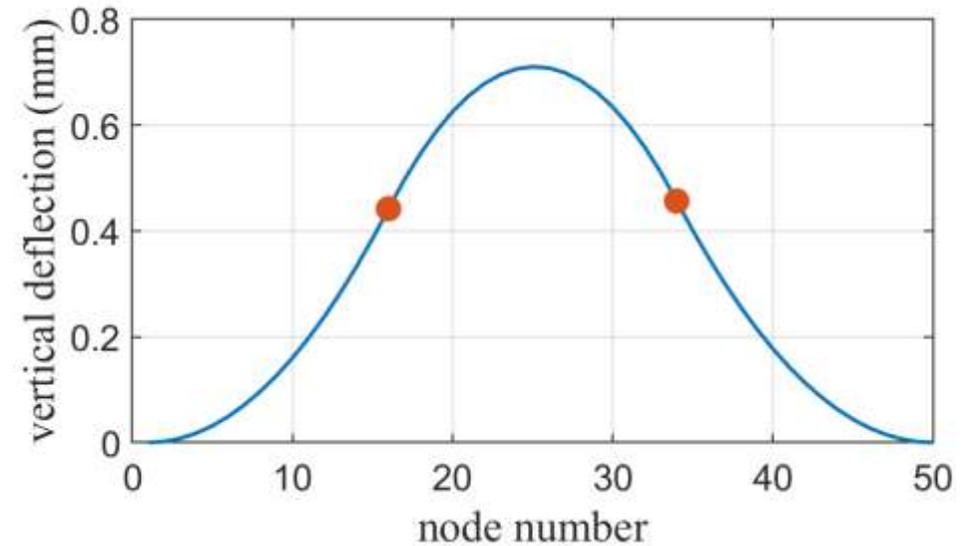
- Convergence study performed using a fixed–fixed beam under a unit midpoint load
- Compared FEM midpoint displacement with the analytical Euler–Bernoulli solution
- 49 elements (50 nodes) achieved  $< 0.1$  % relative error
- 1D FEM captures beam stiffness and curvature behavior efficiently



# RESULTS AND CONCLUSION

# STATIC ANALYSIS

- Validate that the FEM correctly represents bending from localized moment couples.
- Two equal and opposite moments ( $\pm 0.5$  Nm) applied at nodes 16 and 34.
- Induced concave and convex deflection patterns match Euler-Bernoulli beam behavior.
- Confirms that moment-based actuation produces physically consistent curvature.
- This is a necessary step before applying active piezoelectric control.



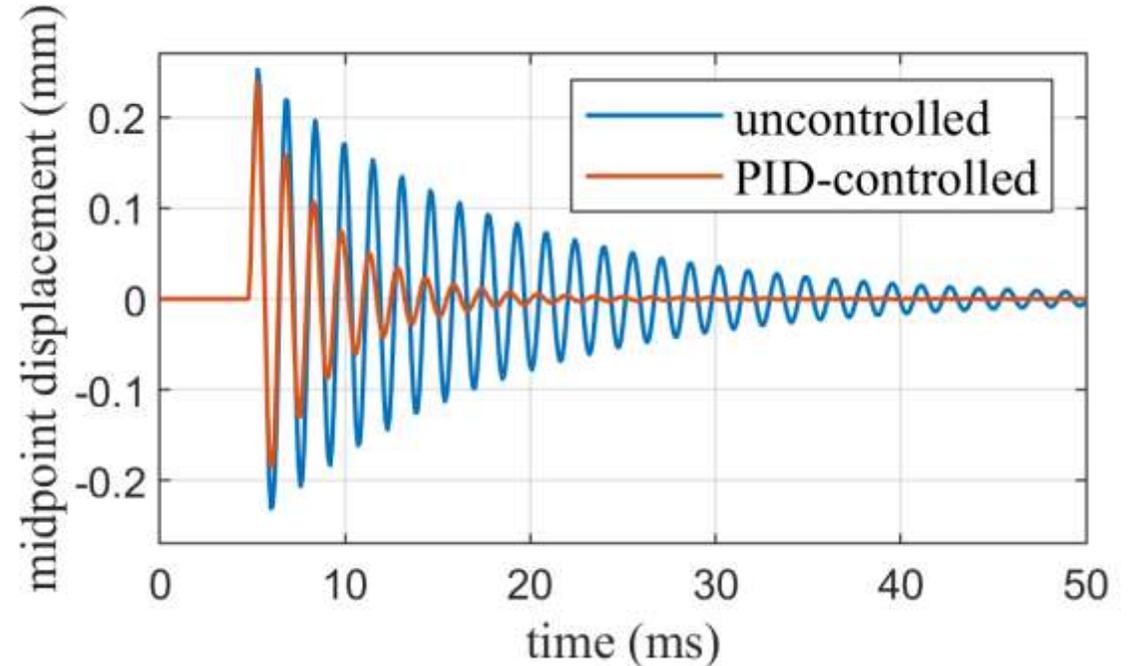
# DYNAMIC ANALYSIS

- Demonstrate dynamic response of the beam under impulsive loading and confirm the effect of simulated actuator moments.
- Beam excited by short-duration impact load (30 N for 0.1 ms).
- Moment-based PID actuation applied at midspan to mimic piezoelectric control:

$$M_{\text{control}} = -K_p \Delta\theta - K_d \Delta\dot{\theta} - K_i \int \Delta\theta dt$$

where  $\Delta\theta = \theta_R - \theta_L$ ,

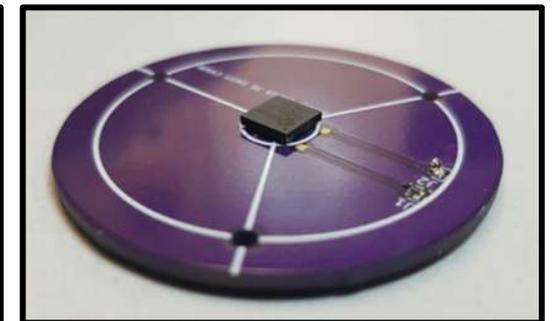
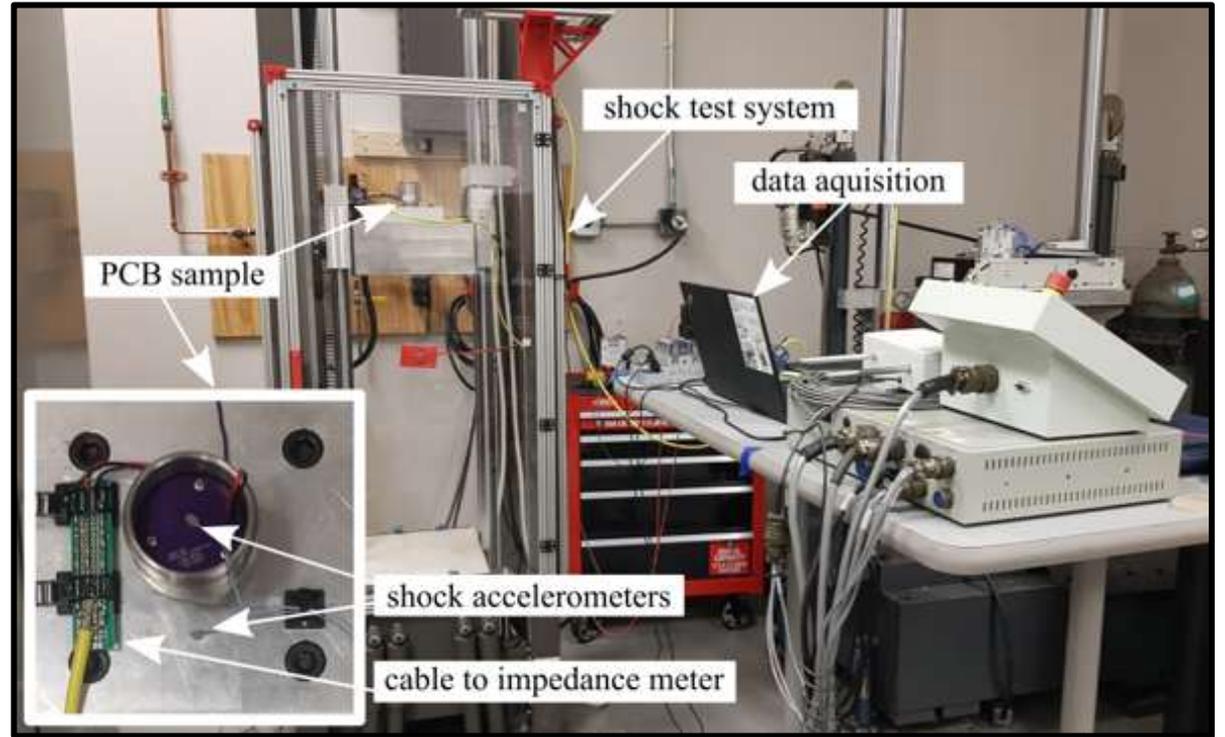
$K_p = 0.25$ ,  $K_d = 5.0e-4$ , and  $K_i = 0.1$ .



metric	improvement
peak displacement	5.09 %
settling time	60.75 %
RMS acceleration	0.25 %

# CONCLUSION

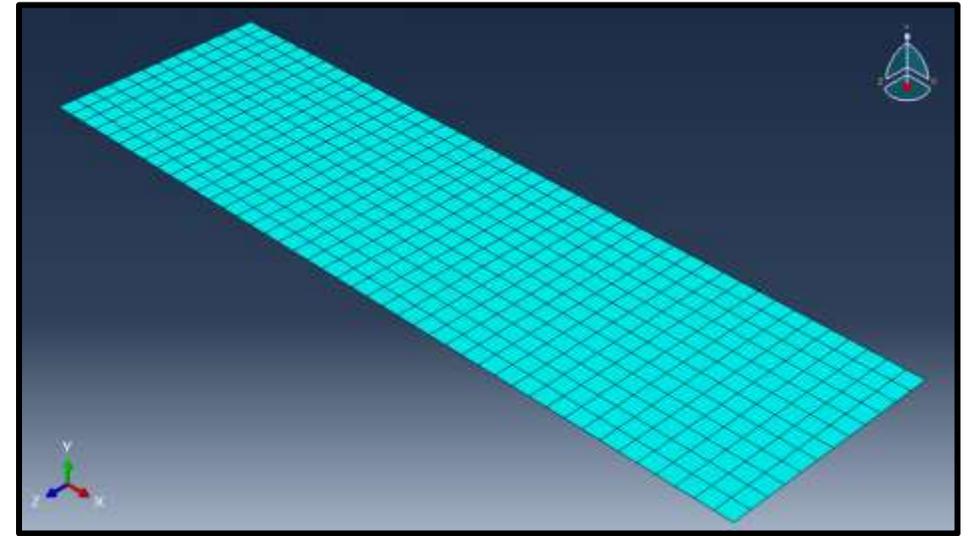
- Shock and impact events demand millisecond response
- Current FEM and control models are too heavy to run in real time
- Full 3D models are impractical for embedded systems
- Need simple, fast FEMs that still capture key physics
- Enables real-time sensing and control at the edge



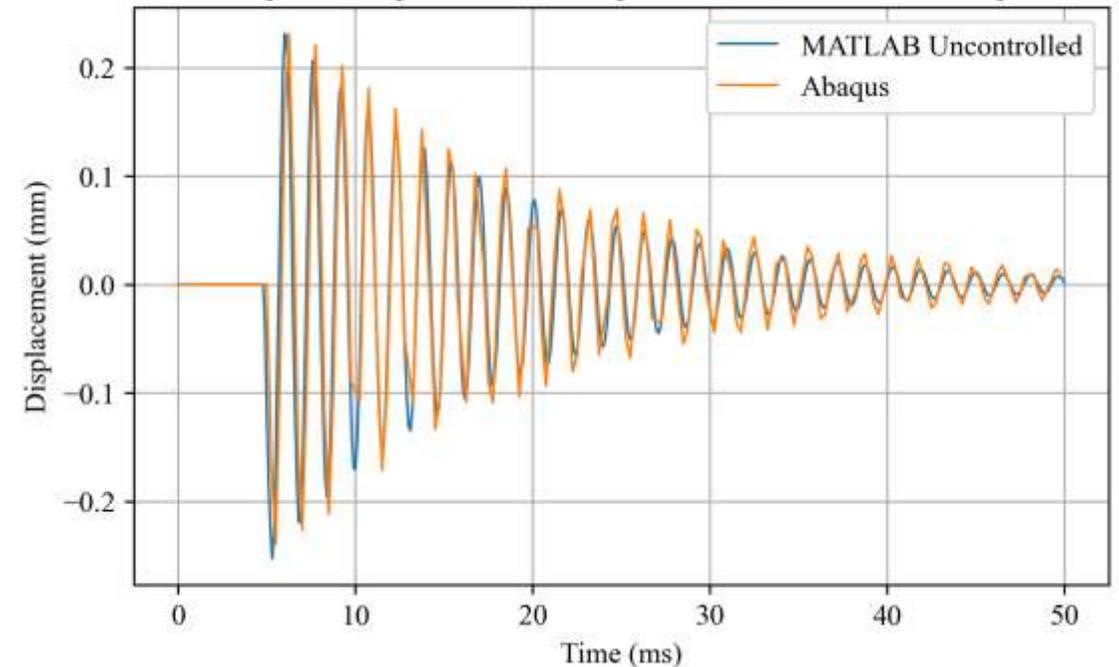
**EXTENDED WORK**

# FEM MODELING

- A higher-fidelity shell model of the fixed–fixed beam was created to mirror the simplified numerical model.
- Both models were subjected to the same transient impact load to compare displacement responses.
- Midspan deflection histories showed strong agreement, confirming that the simplified formulation captures the dominant bending dynamics.

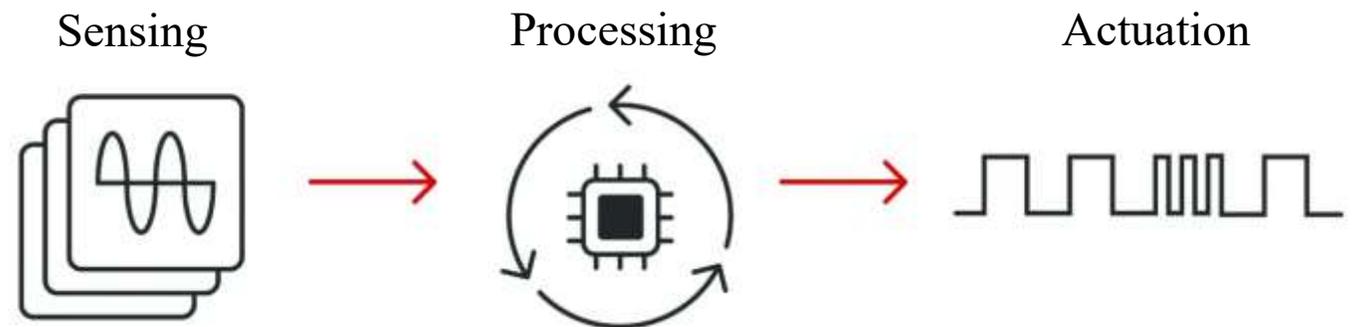
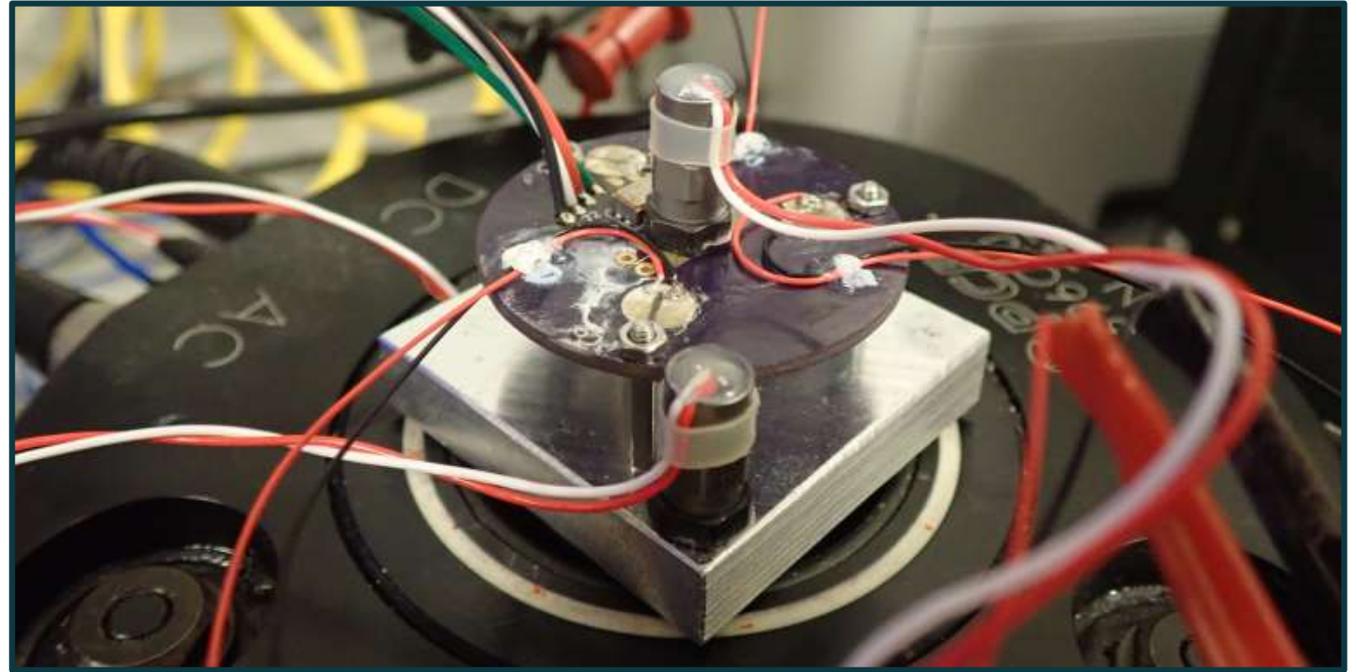


Midpoint Displacement Comparison: MATLAB vs Abaqus



# FUTURE WORK

- Improvement of simulation models.
- Optimization of learning-based control strategies.
- System analysis.
- Dataset collection.
- Real-time FPGA experimentation.
- Piezoelectric sensing/actuation experimentation.



# QUESTIONS?

This material is based upon algorithms supported by the National Science Foundation grant numbers CCF-1937535, CCF-1956071, CCF-2234921, and CPS-2237696. Additional support from the Air Force Office of Scientific Research (AFOSR) through award no. FA9550-21-1-0083. Any opinions, findings, conclusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation or the United States Air Force.



**ADDITIONAL SLIDES**

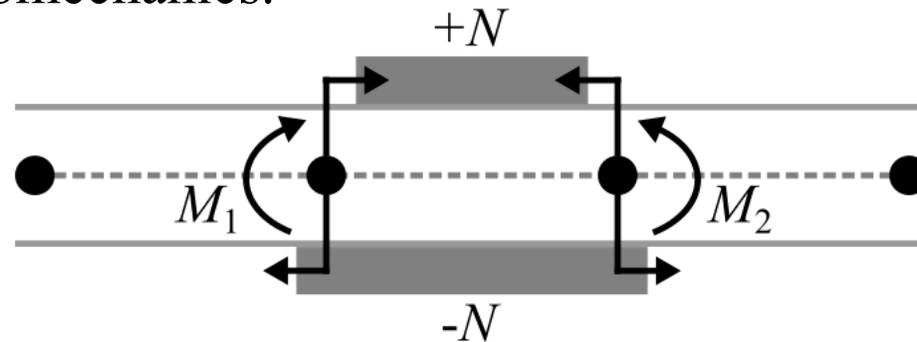
# CONTROL IMPLEMENTATION

- Simulated surface-mounted piezoelectric patches in a 1D FEM framework.
- Each actuator is centered between two nodes along the beam.
- Rather than applying axial forces,  $F_{control}$  is converted into an equivalent moment couple and applied directly to the rotational degrees of freedom at the selected control nodes:

$$M_{control} = F_{control} * \frac{h}{2}$$

where  $h/2$  is the distance from the neutral axis to the actuator surface.

- This approach reproduces the bending effect of symmetric piezo patches without modeling full 3D electromechanics.



# SIMULATION MODEL

- FEM of a fixed-fixed beam subjected to an impact force
- Governed by the **Euler-Bernoulli** Beam Theory:

$$EI \frac{\partial^4 w}{\partial x^4}(x, t) + N \frac{\partial^2 w}{\partial x^2}(x, t) + \rho A \frac{\partial^2 w}{\partial t^2}(x, t) = 0$$

where  $w(x, t)$  is the transverse displacement,  $E$  is Young's modulus,  $I$  is the beam's moment of inertia,  $N$  is the axial force,  $\rho$  is the material density, and  $A$  is the beam's cross-sectional area.

- Discretized into  $n$  nodes; resulting in  $3n$  DOFs, due to axial, transverse, and rotational displacement at each node.
- External forces are applied at a specific node during the impact; control forces are superimposed to represent actuator input.

$$F(t) = F_{impact}(t) + F_{control}(t)$$

# SIMULATION MODEL

- The element stiffness matrix is formed by summing the contributions from axial and bending responses:

$$K_{ij}^{(e)} = \int_0^h EA \phi_i'(x) \phi_j'(x) dx + \int_0^h EI \psi_i''(x) \psi_j''(x) dx$$

where  $\phi_i(x)$  are the linear Lagrange shape functions used for axial deformation and  $\psi_i(x)$  are the cubic Hermite shape functions used for bending deformation.

- The element mass matrix accounts for the inertia associated with both axial and transverse motion:

$$M_{ij}^{(e)} = \int_0^h \rho A \phi_i(x) \phi_j(x) dx + \int_0^h \rho A \psi_i(x) \psi_j(x) dx$$

- For an element of length  $l$ , the shape functions and corresponding nodal degrees of freedom are defined as:

$$W_e = [u_1 \ \omega_1 \ \theta_1 \ u_2 \ \omega_2 \ \theta_2]^T$$

# SIMULATION MODEL

- The **Rayleigh damping** matrix  $C$  is constructed as a linear combination of the mass and stiffness matrices:

$$C = \alpha M + \beta K$$

where  $\alpha$  and  $\beta$  are user-defined damping coefficients.

- The discretized equation of motion is expressed as:

$$M\ddot{W} + C\dot{W} + KW = F(t)$$

- Time integration via **Newmark-beta** method, updating displacements, velocities, and accelerations iteratively:

$$\left( K + \frac{\gamma}{\beta_n \Delta t} C + \frac{1}{\beta_n \Delta t^2} M \right) W_{n+1} = F_{n+1} + \text{previous terms}$$

where  $\beta_n$  and  $\gamma$  are Newmark-beta parameters.